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Exercise Sheet 2
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Exercises "Automata and Formal Languages"

Exercise 2.1

Let A_1 and A_2 be DFAs with states Q_1 and Q_2, respectively. You learnt how to use the product automaton to construct a DFA A such that $\mathcal{L}(A) = \mathcal{L}(A_1) \cup \mathcal{L}(A_2)$, where $A = A_1 \otimes A_2$ using the definition from Exercise 1.4. Show that for every $n \geq 2$ there exist DFAs A_1 and A_2 with $|Q_1|, |Q_2| \geq n$ such that the *minimal* DFA accepting $\mathcal{L}(A_1) \cup \mathcal{L}(A_2)$ has $|Q_1| \cdot |Q_2|$ states.

Exercise 2.2

Show that the following length-preserving relations are regular:

(a) $\{(a^n, b^n) \mid n \geq 0\}$ over $\{a, b\}$.

(b) $R_0 = \{(w\#, w'\#) \mid w, w' \in \{0,1\}^n$ and $w' = ((2 \cdot w) \mod 2^n)$ for $n \geq 2\}$ over $\{0, 1, \#\}$.

(c) $R_1 \cap R_2$, where $R_1, R_2 \subseteq \Sigma^* \times \Sigma^*$ are length-preserving and regular.

Exercise 2.3

Finite state transducers are often defined to be automata over $(\Sigma \cup \{\epsilon\}) \times (\Sigma \cup \{\epsilon\})$ rather than $\Sigma \times \Sigma$, that is, they allow both ϵ inputs and outputs. Given such transducers and the induced regular relations, show the following:

- $\{(w, w') \mid w \leq_{\text{lex}} w'\}$ is regular over every finite alphabet, where $\leq_{\text{lex}}$ denotes the lexicographic order.
- $\{(w, w') \mid w, w' \in \mathcal{L}(1(0+1)^*)$ and $w \leq w'\}$ is regular.
- $\{(a^n b^n, c^n) \mid n \geq 0\}$ over $\{a, b, c\}$ is a non-regular relation.
- Regular relations are *not* closed under intersection.

Exercise 2.4

Let $L = \{w \mid w \in \{0,1\}^n$ for $n \geq 2$ and w is divisible by $3\}$ be a language over $\{0, 1\}$. What is $post_{R_0}(L)$ for R_0 of Exercise 2.2? Construct an automaton accepting $post_{R_0}(L)$ using an automaton accepting L and a transducer accepting R_0.

Exercise 2.5

Let $L \subseteq \Sigma^*$ be a language. The equivalence relation $\sim_L$ is defined as follows: $x \sim_L y$ iff $\forall z \in \Sigma^* : xz \in L \Leftrightarrow yz \in L$. The Myhill-Nerode Theorem states that L is regular iff $\Sigma^* / \sim_L$ is finite, that is, iff the number of equivalence classes of Σ^* with respect to $\sim_L$ is finite.

- Use the Myhill-Nerode Theorem to prove that $\{a^k b^n c^m \mid n = k+m;\ k, m > 0\}$ is not regular.
- Give the Myhill-Nerode equivalence classes of $\{0,1\}^*$ for the language that has the same number of occurrences of the substrings 01 and 10. Construct a minimal DFA accepting this language.