Technische Universität München **Winter term 2008/2009**
Theoretische Informatik **Exercise Sheet 1**
Prof. J. Esparza / Dr.-Ing. J. Kreiker **October 16, 2008**

Exercises "Automata and Formal Languages"

Exercise 1.1

Let r be the regular expression $((0+1)(0+1))^*$ over $\Sigma = \{0,1\}$, where $+$ denotes choice.

- Describe $\mathcal{L}(r)$ in words.
- Give a regular expression r' such that $\mathcal{L}(r') = \overline{\mathcal{L}(r)}$, where $\overline{L} = \Sigma^* \setminus L$.
- *Construct* a regular expression r' such that $\mathcal{L}(r') = \overline{\mathcal{L}(r)}$ using constructions learnt in class.

Exercise 1.2

Let Σ be a finite alphabet. A function $h : \Sigma \to \Sigma^*$ is called a *homomorphism*. We define:

$$\begin{aligned} h(\sigma_1 \ldots \sigma_n) &= h(\sigma_1) \ldots h(\sigma_n) && \text{for } \sigma_1, \ldots, \sigma_n \in \Sigma \\ h(L) &= \{h(w) \mid w \in L\} && \text{for } L \subseteq \Sigma^* \\ h^{-1}(L) &= \{w \mid h(w) \in L\} && \text{for } L \subseteq \Sigma^* \end{aligned}$$

Let $L \subseteq \Sigma^*$ be regular and let $h : \Sigma \to \Sigma^*$ be a homomorphism.

- Show that $h(L)$ regular.
- Show that $h^{-1}(L)$ is regular.
- Use *only* closure properties and the fact that $\{0^n 1^n \mid n \geq 0\}$ is not regular to show that the following languages over $\{0,1,2\}$ are not regular:
  - $\{0^n 1^n 2^m \mid n, m \geq 0\}$
  - $\{0^{2n} 1 2^{3n} \mid n \geq 0\}$
  - $\{w \mid |w|_0 = |w|_2\}$, where $|w|_\sigma$ denotes the number of occurrences of letter σ in word w.

Exercise 1.3

Since regular languages are closed under complement and intersection, we may extend regular expressions with such primitives: If r and r' are regular expressions, then also $\overline{r}$ and $r \cap r'$ are regular expressions. A language $L \subseteq \Sigma^*$ is called *star-free*, iff there exists an extended regular expression r *without* a Kleene star such that $L = \mathcal{L}(r)$. For example, Σ^* is star-free, because it is the same as $\mathcal{L}(\overline{\emptyset})$. Show that $\mathcal{L}((01+10)^*)$ is star-free.

Exercise 1.4

An NFA $(Q, \Sigma, \delta, q_0, F)$ (where $\delta : Q \times \Sigma \to 2^Q$) is called *complete*, iff $|\delta(q,a)| \geq 1$ for all $q \in Q$ and $a \in \Sigma$. We define the *or-product* of two NFAs $A_i = (Q_i, \Sigma, \delta_i, q_{0i}, F_i)$ $(i = 1,2)$ to be $A_1 \otimes A_2 = (Q_1 \times Q_2, \Sigma, \delta, (q_{01}, q_{02}), F)$, where δ is as in $A_1 \times A_2$ and where $F = \{(q_1, q_2) \mid q_1 \in F_i \textbf{ or } q_2 \in F_2\}$.

- Prove: If A_1 and A_2 are complete NFAs, then $\mathcal{L}(A_1 \otimes A_2) = \mathcal{L}(A_1) \cup \mathcal{L}(L_2)$.
- Completeness is thus sufficient for $\mathcal{L}(A_1 \otimes A_2) = \mathcal{L}(A_1) \cup \mathcal{L}(L_2)$. Is it also necessary?