# Existence of home states in Petri nets is decidable

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**Abstract.** We show that the problem whether a given Petri net has a home state (a marking reachable from every reachable marking) is decidable, and at least as hard as the reachability problem.
**Keywords:** Home states, Petri nets, reachability, semilinear sets.

## 1 Introduction

Frequently, dynamic systems must have "home states", which are defined as states that can be reached from whichever state the system might be in. In various electronic devices, home states may be entered automatically after periods of inactivity, or may be forced to be reached by pushing a "reset" button. In self-stabilising systems [3], failure states can be recovered from automatically, preferably ending up in regular, non-erroneous home states. In Markov chain theory, home states are called "essential" states [2] a particularly important class being that of the "recurrent" states.

The main two decision problems concerning home states are (1) given a dynamic system $S$ and a state $q$, is $q$ a home state of $S$ ? and (2) given a system $S$, does it have a home state? We call them the home state problem (HSP) and the home state existence problem (HSEP), respectively.

For finite-state systems, both HSP and HSEP are trivially decidable, but this is no longer true for models which may have an infinite state space, like Petri nets. For Petri net models, HSP (and, in fact, a more general problem) was shown decidable in [5, 6], but our knowledge about HSEP is more limited: the only result was obtained in [1], where it was shown that all live and bounded free-choice nets have home states, while live and bounded asymmetric-choice nets may not. HSEP is explicitly mentioned as an open problem in Wimmel's compilation of open problems in Petri net theory [12].

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In the first part of the paper we show that HSEP is decidable, and provide an algorithm that constructs a home state whenever there is one. The algorithm combines the decision procedure for HSP described in [5] with a more recent result showing that the mutual reachability relation for Petri nets (the relation containing the pairs of markings of a net that are reachable from each other) is effectively semilinear [10]. In the second part of the paper we show that the reachability problem of Petri nets can be reduced polynomially to HSEP. This underlines the hardness of HSEP. In the concluding section of the paper, we mention some related problems whose decidability remains open.

## 2 Basic concepts

We assume familiarity with elementary notions of Petri nets [11], such as the notation $N = (P, T, F)$ for a net with places $P$, transitions $T$, and arcs $F$. The set of $N$'s markings is $\mathbb{N}^P$, and its initial marking (if one exists) is usually denoted by $M_0$. A marking $M' \in \mathbb{N}^P$ is reachable from a marking $M \in \mathbb{N}^P$ by a firing sequence $\tau \in T^*$, also denoted by $M \xrightarrow{\tau} M'$, if $\tau$ leads from $M$ to $M'$. The set of all markings reachable from $M$ is denoted by $[M\rangle$. We assume Petri nets to be finite. Observe that an initially marked finite net $(N, M_0)$ can be unbounded, thus generating an infinite state space (i.e., an infinite set $[M_0\rangle$).

**Definition 1.** *Let $(N, M_0)$ be an initially marked net. A set of markings $\mathcal{M}$ of $N$ is a* home space *of $(N, M_0)$ if for every marking $M$ which is reachable from $M_0$, some marking in $\mathcal{M}$ is reachable from $M$. A marking $M$ is called a* home state *of $(N, M_0)$ if $\{M\}$ is a home space of $(N, M_0)$.*

Observe that a set of markings can be a home space of $(N, M_0)$, without necessarily containing a home state of $(N, M_0)$. Also observe that $\emptyset$ is never a home space while $[M_0\rangle$ always is.

**Definition 2.** *Let $N$ be a net. Two markings $M, M'$ of $N$ are* mutually reachable *if $M'$ is reachable from $M$ and vice versa. The* mutual reachability relation *of $N$ is the set containing the pairs $(M, M')$ of markings of $N$ such that $M$ and $M'$ are mutually reachable.*

Note that this defines an equivalence on $\mathbb{N}^P$ which does not depend on any initial marking, but only on the structure of $N$.

**Definition 3.** *Let $N$ be a net. A marking $M$ of $N$ is a* bottom marking *of $N$ if for every marking $M'$ reachable from $M$, the markings $M$ and $M'$ are mutually reachable.*

Note that a dead marking (enabling no transition) is automatically a bottom marking. Also, observe that bottom markings of $N$ are related to home states of a marked net $(N, M_0)$, with the same underlying net $N$. If $M$ is a home state of a marked Petri net $(N, M_0)$, then $M$ is reachable from $M_0$, and it is a bottom marking of $N$. In that case, any other bottom marking reachable from $[M_0\rangle$ is also a home state. However, a marking $M$ can be reachable from $M_0$ and also be a bottom marking of $N$, without there necessarily being a home state of $(N, M_0)$.

The bottom markings of $N$ are computationally more amenable than the home states of $(N, M_0)$ because (as it will turn out) they are semilinear in the following sense.

**Definition 4.** *Let $k \in \mathbb{N}$. A set $\mathcal{M} \subseteq \mathbb{N}^k$ is* linear *if there exists a root vector $\rho \in \mathbb{N}^k$ and a finite set of periods $\Pi = \{\pi_1, \ldots, \pi_n\} \subseteq \mathbb{N}^k$ such that*

$$\mathcal{M} \;=\; \bigcup_{\lambda_1,\ldots,\lambda_n \in \mathbb{N}} \{M \in \mathbb{N}^k \mid M = \rho + \sum_{i=1}^{n} \lambda_i \pi_i\}$$

*and* semilinear *if $\mathcal{M} = \mathcal{M}_1 \cup \ldots \cup \mathcal{M}_m$ for $m$ linear sets $\mathcal{M}_1, \ldots, \mathcal{M}_m$.*
*We denote $(\rho; \Pi)$ the linear set with root vector $\rho$ and period set $\Pi$.*

A subset of $\mathbb{N}^k$ (for some $k \in \mathbb{N}$) is semilinear if and only if it is Presburger definable [8]. Semilinearity and Presburger definability extend to subsets of $\mathbb{N}^k \times \mathbb{N}^k$ using $\mathbb{N}^k \times \mathbb{N}^k = \mathbb{N}^{2k}$.

A set $\mathcal{M}$ of markings of a net $N$ with $k$ places is *effectively semilinear* if there is an algorithm that on input $N$ returns root vectors $\rho_1, \ldots, \rho_m \in \mathbb{N}^k$ and period sets $\Pi_1, \ldots, \Pi_m \subseteq \mathbb{N}^k$ such that $\mathcal{M} = \bigcup_{i=1}^{n} (\rho_i; \Pi_i)$. Similarly, $\mathcal{M}$ is *effectively definable in Presburger arithmetic* if there is an algorithm that on input $N$ returns a formula of Presburger arithemtic defining $\mathcal{M}$. Effectively semilinear and Presburger definable relations on the markings of $N$ are defined analogously, by identifying $\mathbb{N}^k \times \mathbb{N}^k$ with $\mathbb{N}^{2k}$. By [8], effective semilinearity and effective definability in Presburger arithmetic coincide.

The home state existence problem is defined as follows:

HSEP: **Given:** An initially marked Petri net $(N, M_0)$.
**Decide:** Is there a home state $M$ of $(N, M_0)$ ?

## 3 Decidability of HSEP

To commence the proof of the decidability of HSEP, we recall the following strong result by Leroux:

**Theorem 1 ([10]).** *For every Petri net $N$, the mutual reachability relation is effectively definable in Presburger arithmetic*[1].

[1] Observe the emphasis on *effective* definability. Definability in Presburger arithmetic follows from a result by Eilenberg and Sch́utzenberger on commutative monoids [7].

So, by [8], the mutual reachability relation is semilinear. This easily leads to the following result, already described in [4]. Since the proof is short, we give a sketch.

**Theorem 2 ([4]).** *Let $N$ be a net. The set of bottom markings of $N$ is effectively semilinear.*

*Proof.* We show that the predicate $\mathtt{B}(M)$ associated to the set of bottom markings is effectively definable in Presburger arithmetic, and so semilinear. By Theorem 1, we can compute a Presburger predicate $\mathtt{MR}(M, M')$ associated to the mutual reachability relation. Now, we observe – using induction on the length of a firing sequence – that $M$ is a bottom marking iff for every marking $M'$ such that $M$ and $M'$ are mutually reachable, and for every $M''$ such that there is some $t \in T$ with $M' \xrightarrow{t} M''$, the markings $M$ and $M''$ are also mutually reachable. Hence

$$\mathtt{B}(M) \iff \forall M' \forall M'' \colon (\mathtt{MR}(M, M') \wedge (\exists t \in T \colon M' \xrightarrow{t} M'')) \Rightarrow \mathtt{MR}(M, M'')$$

and we are done, since the single-step reachability relation can be expressed semilinearly, and Presburger formulas are stable over conjunction, disjunction, implication, and universal quantification [8]. □

Now consider an initially marked Petri net $(N, M_0)$. If its home states coincide with the set of bottom markings reachable from $[M_0\rangle$, then Theorem 2 already leads to an algorithm, because semilinear set reachability is decidable [9]. However, the set of home states of $(N, M_0)$ could be a proper subset of the set of reachable bottom states. Hence we need a further step in the development of an algorithm, which utilises the decidability result of [5].

**Theorem 3 ([5]).** *Let $(N, M_0)$ be a Petri net and let $\mathcal{M}$ be a linear set of markings of $N$. It is decidable if $\mathcal{M}$ is a home space of $(N, M_0)$.*

This theorem allows us to define the algorithm of Figure 1. We prove that it indeed solves HSEP.

**Theorem 4.** *The algorithm of Figure 1 decides if a Petri net $(N, M_0)$ has a home state.*

*Proof.* We first show that the algorithm is indeed an algorithm. The sets $B$, $B_1$, …, $B_n$ are effectively computable by Theorem 2 and the fact that semilinear sets are finite unions of linear sets. Since $B_j$ and $\{M_0'\}$ are linear sets, the function calls *homespace*$(B_j)$ and *homespace*$(\{M_0'\})$ in lines 3 and 5 are effectively computable by Theorem 3. Finally, the marking $M_0'$ at line 4 can be found by executing all the firing sequences of $(N, M_0)$ of length 0, 1, 2, etc., until one of them leads to a marking of $B_j$. Observe that the procedure terminates because, by line 3, the set $B_j$ is a home space of $(N, M_0)$, and so some marking of $B_j$ is indeed reachable

/* Algorithm to decide if $(N, M_0)$ has a home state. */
/* The function *homespace*$(\mathcal{M})$ returns **true** if $\mathcal{M}$ */
/* is a home space of $(N, M_0)$, and **false** otherwise. */

1: compute the semilinear set $B$ of bottom markings of $N$, and linear sets $B_1, \ldots, B_m$ such that $B = B_1 \cup \ldots \cup B_m$
2: **for** $j = 1, \ldots, m$ **do**
3:     **if** *homespace*$(B_j)$ **then**
4:         pick a marking $M'_0 \in [M_0\rangle \cap B_j$
5:         **if** *homespace*$(\{M'_0\})$ **then**
6:             output " $M'_0$ is a home state of $(N, M_0)$ " and **halt**
7:         **else**
8:             output " $(N, M_0)$ has no home state " and **halt**
9:         **end if**
10:     **end if**
11: **end for**
12: output " $(N, M_0)$ has no home state "

**Fig. 1.** Algorithm to check the existence of home states.

from $M_0$. To check that the marking reached by a firing sequence belongs to $B_j$ reduces to solving a set of diophantine equations: indeed, if the marking is $M$, the root of $B_j$ is $\rho_j$, and the periods of $B_j$ are $\pi_{1j}, \ldots, \pi_{n_j j}$, then the problem reduces to deciding if the equation $M = \rho_j + \sum_{i=1}^{n_j} \lambda_i \pi_{ij}$ has a solution over the natural numbers.

We now prove that the algorithm is correct. Termination is clear. Partial correctness is a consequence of the following three claims:

*Claim 1*: If the algorithm terminates at line 12, then $(N, M_0)$ has no home state.

We prove the contrapositive. Assume $(N, M_0)$ has a home state $M$. By the definition of a home state, $M$ is a bottom marking of $N$, and so it belongs to at least one of $B_1, \ldots, B_m$, say $B_j$. It follows that $B_j$ is a home space of $N$. Since the guard at line 3 evaluates to **true** for $B_j$, the algorithm does not terminate at line 12.

*Claim 2*: If the algorithm terminates at line 8, then $(N, M_0)$ has no home state.

If the algorithm terminates at line 8, then, because of lines 3 and 5, the marking $M'_0 \in [M_0\rangle \cap B_j$ is not a home state of $(N, M_0)$. Since $M'_0$ is not a home state, there is some marking $M_1 \in [M_0\rangle$ such that $M'_0 \notin [M_1\rangle$. Since $M'_0 \in B_j$, $M'_0$ is a bottom marking. This, together with $M'_0 \notin [M_1\rangle$, implies $M_1 \notin [M'_0\rangle$. Hence $M'_0$ and $M_1$ are not reachable from each other, but both are reachable from $M_0$. Altogether, this implies that $(N, M_0)$ has no home state.

*Claim 3*: If the algorithm terminates at line 6, then $M'_0$ is a home state of $(N, M_0)$.

If the algorithm terminates at line 6 then $\{M'_0\}$ is a home space of $(N, M_0)$ which, by the definition of a home space, is the same as $M'_0$ being a home state of $(N, M_0)$. □

## 4 Hardness of HSEP

We show that the existence of home states is computationally at least as hard as the reachability problem for Petri nets. Recall that the reachability problem can be reduced in polynomial time to the single-place zero-marking reachability problem SPZR [9], which is defined as follows:

SPZR: $\begin{cases} \textbf{Given:} & \text{An initially marked Petri net } (N, M_0)\text{, a place } p \text{ of } N. \\ \textbf{Decide:} & \text{Is there a reachable marking } M \text{ such that } M(p) = 0\text{ ?} \end{cases}$

So it suffices to exhibit a polynomial reduction from SPZR to the existence of home states. Given a Petri net $(N, M_0)$ and a place $p$ of $N$, we construct in polynomial time a Petri net $(\tilde{N}, \tilde{M}_0)$ in two steps as follows (see Figure 2):

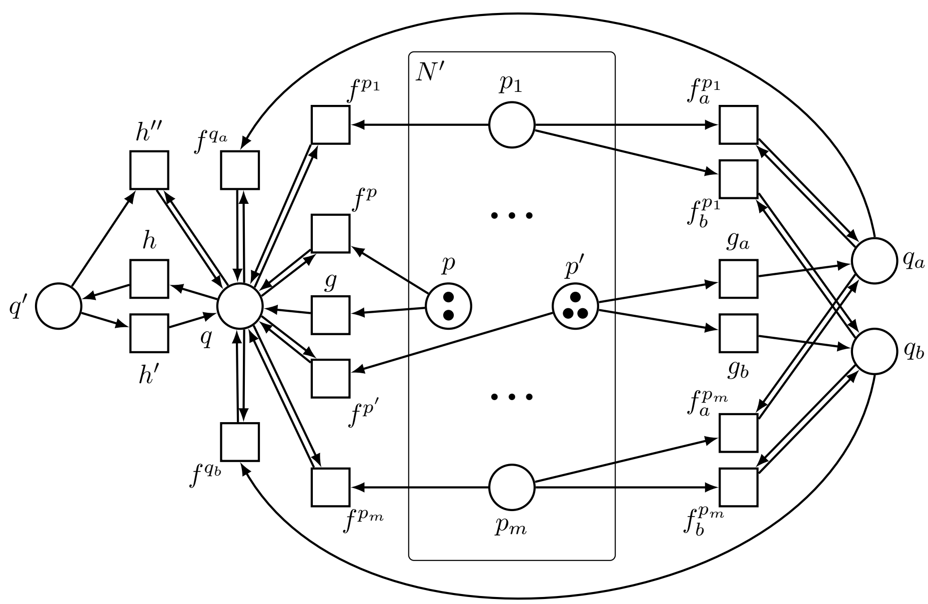


**Fig. 2.** The Petri net $(\tilde{N}, \tilde{M}_0)$.

**Step 1:** Construct $(N', M_0')$ from $(N, M_0)$ by adding a single new place $p'$ with the same input and output transitions as $p$, but with one more token, i.e.,

$M_0'(p') = M_0(p) + 1$. Clearly, we have:

$$
\begin{aligned}
& (\exists M \in [M_0\rangle \colon M(p) = 0 \text{ in } (N, M_0)) \\
\iff & (\exists M' \in [M_0'\rangle \colon M'(p) = 0 \land M'(p') = 1 \text{ in } (N', M_0'))
\end{aligned}
$$

**Step 2:** Construct $(\tilde{N}, \tilde{M}_0)$ from $(N', M_0')$ in the following way:

- Add three new places $q, q_a, q_b$ carrying no tokens in the initial marking, and three new "goto" transitions $g, g_a, g_b$.
- Add arcs $(p, g), (g, q)$, $(p', g_a), (g_a, q_a)$ and $(p', g_b), (g_b, q_b)$.
  Observe: tokens in $p$ can move to $q$ at any time, and tokens in $p'$ can move to $q_a$ or $q_b$ at any time.
- Add three families of "flush" transitions, $f^r, f^s_a, f^s_b$, for $r \in P' \cup \{q_a, q_b\}$ and $s \in P' \setminus \{p, p'\}$, where $P'$ denotes the set of places of $N'$ (in Figure 2 $P' = \{p_1, \ldots, p_m\}$, including $p$ and $p'$).
- Add an arc from every place $r$ in $P' \cup \{q_a, q_b\}$ to $f^r$, and arcs $(q, f^r), (f^r, q)$.
  Observe: from any marking with at least one token in $q$, the $f^r$ transitions can "flush" all other tokens in $P' \cup \{q_a, q_b\}$ and lead to a marking with tokens only in place $q$.
- Add an arc from every place $s$ in $P' \setminus \{p, p'\}$ to $f^s_a$, and to $f^s_b$, and arcs $(q_a, f^s_a)$, $(f^s_a, q_a)$ and $(q_b, f^s_b)$, $(f^s_b, q_b)$.
  Observe: from any marking with at least one token in $q_a$ or $q_b$, the $f^s_a$ or $f^s_b$ transitions can "flush" tokens, leading to a marking with tokens only (possibly) in $p$, $p'$, $q$, $q_a$, and $q_b$.
- Add a place $q'$, three transitions $h, h', h''$, and arcs $(q, h)$, $(h, q')$, $(q', h')$, $(h', q)$, $(q, h'')$, $(h'', q)$, $(q', h'')$.
  Observe: from any marking with tokens only in $q$ we can fire the transitions $h, h', h''$ to reach a marking with exactly one token in $q$ and no token on $q'$.

**Proposition 1.** *Let $(N, M_0)$ be a Petri net with a place $p$, and let $(\tilde{N}, \tilde{M}_0)$ be the Petri net defined above. Then:*

$$(\exists M \in [M_0\rangle \colon M(p) = 0) \iff (\tilde{N}, \tilde{M}_0) \text{ has no home state.}$$

*Proof.* By the property of the net $(N', M_0')$ obtained after Step 1 it suffices to show:

$$
\begin{aligned}
& (\exists M' \in [M_0'\rangle \colon M'(p) = 0 \land M'(p') = 1 \text{ in } (N', M_0')) \\
\iff & (\tilde{N}, \tilde{M}_0) \text{ has no home state}
\end{aligned}
$$

($\Rightarrow$): Suppose $M'$ is reachable in $(N', M_0')$ and puts 1 token on $p'$ and 0 tokens on $p$. Let $\tilde{M}$ be the marking of $\tilde{N}$ that coincides with $M'$ on $N'$, and puts zero tokens on the additional places $q, q', q_a, q_b$. Then $\tilde{M}$ is reachable from $\tilde{M}_0$ in $\tilde{N}$, and it enables both $g_a$ and $g_b$. Let $\tilde{M}[g_a\rangle\tilde{M}_a$ and $\tilde{M}[g_b\rangle\tilde{M}_b$; then $\tilde{M}_a$ puts one token in $q_a$, and no tokens in $q_b, q, q', p, p'$, and $\tilde{M}_b$ puts one token in $q_b$, and no tokens in $q_a, q, q', p, p'$. From $\tilde{M}_a$ (or from $\tilde{M}_b$), we can now exhaustively fire the flush transitions $f_a^s$ (or $f_b^s$). This leads to two distinct deadlock markings having exactly one token on $q_a$ or on $q_b$, and no tokens elsewhere; thus $(\tilde{N}, \tilde{M}_0)$ has no home state.

($\Leftarrow$): Suppose that *no* reachable marking of $(N', M'_0)$ puts 1 token on $p'$ and 0 tokens on $p$. Since, by construction of $N'$, every reachable marking $M'$ of $(N', M'_0)$ satisfies $M'(p') = M'(p) + 1$, we deduce that no reachable marking of $(N', M'_0)$ puts 0 tokens on $p$. We claim that every reachable marking $\tilde{M}$ of $(\tilde{N}, \tilde{M}_0)$ satisfies $\tilde{M}(p) + \tilde{M}(q) + \tilde{M}(q') \geq 1$. Let $\sigma$ be an occurrence sequence such that $\tilde{M}_0 \xrightarrow{\sigma} \tilde{M}$. Then $\sigma = \sigma_1 t_1 \sigma_2 t_2 \ldots \sigma_n t_n \sigma_{n+1}$, where $\sigma_1, \ldots, \sigma_{n+1}$ are (possibly empty) sequences of transitions of $N'$, and $t_1, \ldots, t_n$ are occurrences of $g$, $g_a, g_b$, or of "flush" transitions. If $t_i = g$ for some $i \in \{1, \ldots, n\}$, then the marking reached after firing $t_i$ puts at least one token in $q$. Since, by construction, every output transition of $\{q, q'\}$ is also an input transition of $\{q, q'\}$, the sum of the number of tokens in $q$ and $q'$ cannot become 0 in any later marking, and so $\tilde{M}(q) + \tilde{M}(q') \geq 1$. If $t_i \neq g$ for every $i \in \{1, \ldots, n\}$, then, since $t_1, \ldots, t_n$ can only remove tokens from the places of $P' \cup \{p, p'\}$, never adding any, $\sigma' = \sigma_1 \sigma_2 \ldots \sigma_n \sigma_{n+1}$ is also an occurrence sequence of $(N', M'_0)$. Let $\tilde{M}_0 \xrightarrow{\sigma'} \tilde{M}'$. Since $\tilde{M}'$ is a reachable marking of $(N', M'_0)$, we have $\tilde{M}'(p) > 0$, and, since the transitions $t_a, \ldots, t_n$ do not remove tokens from $p$, we also have $\tilde{M}(p) > 0$, and the claim is proved.

Since every reachable marking $\tilde{M}$ of $(\tilde{N}, \tilde{M}_0)$ satisfies $\tilde{M}(p) + \tilde{M}(q) + \tilde{M}(q') \geq 1$, from any reachable marking we can fire the flush transitions to reach a marking putting only tokens in $q$ (observe that the family $f^s$ also contains transitions to remove tokens from $q_a$ and $q_b$). From any such marking we can then fire the transitions $h, h'$ to reach the marking that puts only a single token remains on $q$, and no tokens elsewhere. So this marking is a home state. □

## 5 Concluding remarks

This paper has answered, in the positive, one of several still open decidability questions in Petri net theory. Amongst the unknown ones, the structural liveness problem is defined as follows:

$$\text{STLP:} \quad \begin{cases} \textbf{Given:} & \text{A Petri net } N. \\ \textbf{Decide:} & \text{Is there a marking } M_0 \text{ of } N \text{ such that } (N, M_0) \text{ is live ?} \end{cases}$$

and the partial home state existence problem is defined as follows:

$$\text{pHSEP:} \quad \begin{cases} \textbf{Given:} & \text{A Petri net } (P, T, F) \text{, a set } \tilde{P} \subseteq P \text{, a marking } \tilde{M} \in \mathbb{N}^{\tilde{P}}. \\ \textbf{Decide:} & \exists M \in \mathbb{N}^P, \tilde{M}' \in \mathbb{N}^{\tilde{P}} \text{ such that } M|_{\tilde{P}} = \tilde{M} \\ & \quad \text{and } \forall M'' \in [M\rangle \, \exists M' \in [M''\rangle : \; M'|_{\tilde{P}} = \tilde{M}' \text{ ?} \end{cases}$$

A polynomial reduction from STLP to pHSEP is given in [12], but it is not entirely strict. Also, it is not known whether these two problems are decidable or not. It remains to be investigated whether the algorithm described in the present paper can be extended to pHSEP, and if so, whether all of this could be used in order to resolve the decidability status of STLP.

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