On the Analysis and Synthesis of Free Choice Systems

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Abstract: This invited paper present in a semi-formal illustrative way several new results concerning the analysis and synthesis of free choice systems. It is a complementary work of the survey by E. Best [Best 87]. In the *analysis* part, we characterize liveness and boundedness in *linear algebraic* terms. As a consequence of the new characterizations, both properties are shown to be decidable (as a whole) in polynomial time. We also provide two different kits of sound and complete *reduction rules* (the one reverse-dual of the other).

We address then the problem of synthezising live and bounded free choice systems within the two basic design methodologies: *top-down* and *modular* (synthesis by *composition of modules*). Two complete kits of top-down synthesis rules are provided. They are essentially the reduction kits obtained before, but this time considered in the reverse direction. The completeness of the kits can be used to prove new results (or give new proofs of old results) using *structural induction* on the chain of applications of the rules that synthezise a given system. In the modular approach, exact conditions for the preservation of liveness and boundednes under compositions of systems are given. These conditions are the absence of certain design errors, called *killing choices*, *killing joints*, *synchronic mismatches* and *state mismatches.* They help to understand why a certain system is not well behaved.

Keywords: Analysis, free choice nets, linear algebra techniques, reduction, state refinement, structure of systems, modular synthesis, top-down synthesis, transformation.

CONTENTS

1 Introduction

Petri Nets (PNs) are well known abstract models of concurrent systems, with an intuitively appealing graphical representation, very appreciated in engineering circles. They provide a formal frame where the two basic problems, *analysis* and *synthesis*, can be investigated.

The analysis problem can be stated as follows: given a model (a PN model in our case), does it satisfy a certain set of properties of good behaviour? The indigenous PN techniques developed for this problem can be classified into three groups: *reachability, reduction* and *structural* techniques. In systems with a finite number of states (i.e. bounded systems), the reachability approach permits to answer all analysis questions. However, this technique requires an exhaustive exploration of the state space, which hinders its application to large systems. In non-bounded systems only some analysis questions can be answered (e.g. regularity problems [Fink 90]).

Structural techniques are based on the relationships between the behaviour of a system and the structure of its underlying net. More precisely, given a behavioural property, the structural approach tries to find structural properties characterizing it partially (only necessary or sufficient conditions) or totally (necessary and sufficient conditions). Structural analysis techniques use basically *graph theory* (e.g. [Best 87, TV 84]) and *linear algebra/convex geometry* arguments (e.g. [CCS 90b, CS 89a, CS 89b, Laut 87a, MR 80]).

Reduction (or abstraction) techniques [Bert 87, Silv 85] simplify the system by means of *reduction rules* which *preserve* the properties under study (the reduced systems enjoys the property if and only if the original one enjoys it as well). Applying reduction rules in an iterative way, a sequence of progressively more simple models is obtained, in which it is easier to check if the desired property holds. Sometimes the final system is trivial, and the question can be immediately answered; otherwise other analysis techniques are needed.

Synthesis is the second basic, and more difficult, problem. It can be stated as follows: given a set of properties of good behaviour, how to construct systems enjoying them? This problem is strongly related to *design methodologies*. The two basic and complementary approaches are the *top/down* (sometimes *refinement*) and the *modular* (or *compositional*). The first one is just the reverse of the reduction analysis approach. In the second case modules (subsystems) are merged (composed) into new systems.

Petri nets permit to combine in easy and powerful ways three fundamental situations: *sequence*, *conflict* (*choice*) and *concurrency*. The interplay of the last two situations can make it very difficult to find relationships between behaviour and structure. To obtain structural characterizations of behavioural properties, the only actual possibility is to restrict the class of nets in such a way that the interplay between concurrency and choice is particularly simple. The analysis and synthesis problems are trivial for systems in which synchronization is structurally forbidden such as *State Machines*. For *Marked graphs*, systems in which choices are structurally forbidden, they are not so trivial but have been both extensively studied (see [CHEP 71, ES 89b, GL 73, Mura 89]), and are very well understood.

Free Choice systems are located at an interesting place in the tradeoff between *practical modeling power* and *analyzability*. An ordinary net (i.e. arc weights equal to 1) is free-choice iff all the transitions in a conflict have only one input place. This way, choices cannot be influenced by the environment (a concept similar to the internal nondeterminism of TCSP). Partially based on works by Commoner, Hack's thesis [Hack 72] is the pioneer reference for FC nets theory. Two surveys on the results obtained till 1987 are [Best 87, BT 87]. Since then, further contributions are [BCDE 90, BD 90, Des 90, DE 90, ES 89a, ES 89b, Espa 90a, Espa 90b, ES 90a, ES 90b, ES 90c, Vogl 89]. To maintain this work at a semiformal illustrative level, the analysis works surveyed concern only two basic properties: *liveness* and *boundedness*. Analogously, only the synthesis of live and bounded free choice (LBFC) systems is considered. In [Des 90] the reduction/synthesis of live and *safe* FC systems *without frozen tokens* is done using a kit of four rules. Result concerning *home states* in LBFC systems are reported in [BCDE 90, Vogl 89], while the *reachability problem in the home space* for LBFC systems is solved in [DE 90]. The main topics selected for presentation in this paper are the three following:

(1) liveness and boundedness (as a whole) can be linearly characterized for FC systems.

(2) the class of LBFC systems can be reduced/top-down synthetized by means of kits of two rules (one non local).

(3) the class of LBFC systems can be synthetized (and also reduced) by means of modular compositions.

Given the above selection, the concepts of *handles* and *bridges* [ES 89a] are not considered here, basically because they are specially useful for proof techniques, that provide results on LBFC systems that are interpretable at higher-level [ES 90a, ES 90b].

Sections 2 and 3 are devoted to the analysis problem. Section 2 introduces our linear algebraic characterization of liveness and boundedness. It is shown how Hack's duality theorem can be derived from it. Section 3 introduces the kits of reduction rules.

The synthesis of LBFC systems is considered in sections 4 and 5. Two reverse-dual kits of top-down rules are introduced. They are, informally speaking, the reverse of those of reduction rules. The completeness of the kits permits to state a *generative definition* for LBFC systems: those that can be generated by them. Using this alternative definition new results or new proofs of known results can be given using structural induction on the chains of applications of the rules.

We present then exact conditions for the preservation of liveness and boundedness under *synchronisations* of nets (a particular kind of composition in which, essentially, transitions are merged). They are the absence of two design errors called *killing choices* and *synchronic mistmachs*. Using the duality theorem, it is shown that the absence of other two errors (*killing joints* and *state mismatches*) characterize the preservation of liveness and boundedness under *fusions* (compositions in which, essentially, places are merged).

In the sequel $N = (P, T, F)$ is a net, where P represent the set of places, T the set of transitions and F is the flow relation. A marked net or system, (N, M_0), is obtained by associating an initial marking, M_0, to the net N: usually N is said to be the underlying net of the system (N, M_0).

2 A linear algebraic approach to the analysis problem

Our linear algebraic characterization of liveness and boundedness is splitted into two parts. We characterize first the *structure* of the FC nets which can be endowed with a live and bounded marking. The second part characterizes the markings that make such a lively and boundedly markable net live and bounded.

2.1 Some definitions and results

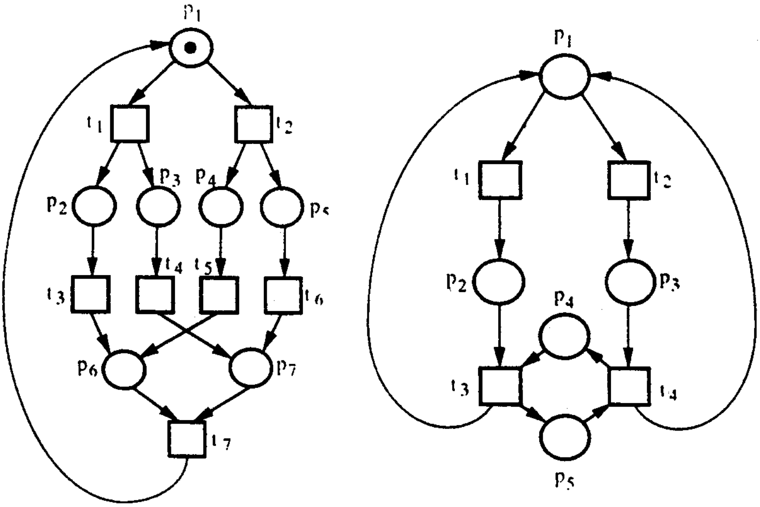
Let us recall structural boundedness and structural liveness notions.

A net N is said to be *structurally bounded* (SB) iff for every initial marking M_0, the system $\langle N, M_0\rangle$ is bounded. The interest of structural boundedness is that it does not depend on any initial marking, but only on the underlying net N.

A net N is *structurally live* (SL) iff there exists at least one initial marking, M_0, for which $\langle N, M_0\rangle$ is live. Structural liveness is a necessary condition for liveness. Once again, structural liveness depens only on the net N.

Our first result, in the next section, characterizes structural liveness and structural boundedness (SL&SB). This is not in general what we promised above, since not every net that can be endowed with a live and bounded marking is SL&SB (although the converse obviously holds). But the following result, a consequence of classical results [Hack 72], shows that both notions collapse for FC nets.

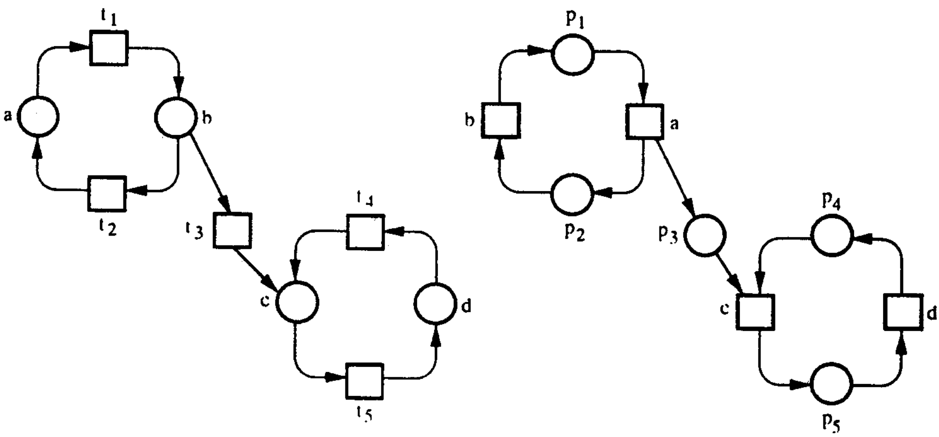
Proposition 2.1 **[Espa 90b]** *Let N be an FC net. Then, there exists M_0 such that $\langle N, M_0\rangle$ is live and bounded iff N is SL&SB.*



(a) Structurally live (r=ρ)
r = rank(C) = 5
ρ= m+n-a-1=7+7-8-1=5

(b) Structurally non live (r≠ρ)
r = rank(C) = 3
ρ= m+n-a-1=4+5-6-1=2

Figure 2.1. Two consistent and conservative free choice nets.



(a) Non consistent state machine:
Structurally non live, even if r = ρ
r = rank(C)=3
ρ= m+n-a-1=5+4-5-1=3

(b) Non conservative marked graph:
Structurally unbounded, even if r = ρ
r = rank(C)=3
ρ= m+n-a-1=4+5-5-1=3

Figure 2.2. Non consistency or non-conservativeness destroy the algebraic characterization of theorem 2.3.

The characterization we provide of SL&SB is given in linear algebraic terms. It complements for FC nets the following well known result, in which SL&SB are related to two properties of the incidence matrix, called *conservativeness* and *consistency*. A net $N = (P, T, F)$ with incidence matrix C is *conservative* iff ($Y > 0$ means $Y(p) > 0$, $\forall p \in P$):

$$\exists Y > 0: Y^T \cdot C = 0$$

N is *consistent* iff ($X > 0$ means $X(t) > 0$, $\forall t \in T$):

$$\exists X > 0: C \cdot X = 0$$

Conservativeness and consistency can be stated in term of *p*- and *t-semiflows*, respectively. A rational-valued vector $Y \gneqq 0$ ($X \gneqq 0$) is a p-semiflow (t-semiflow) of $N = (P, T, F)$ iff $Y^T \cdot C = 0$ ($C \cdot X = 0$). The *support* of a p-semiflow Y, denoted by $\|Y\|$, is the set $\|Y\| = \{p \in P : Y(p) > 0\}$. Analogously, $\|X\| = \{t \in T : X(t) > 0\}$ is the support of the t-semiflow X. Therefore:

$$\begin{array}{lcl} N \text{ is conservative} & \Leftrightarrow & \exists Y, \text{ a p-semiflow such that } \|Y\| = P \\ N \text{ is consistent} & \Leftrightarrow & \exists X, \text{ a t-semiflow such that } \|X\| = T \end{array}$$

Theorem 2.2 [Sifa 78, MR 80] *Let N be an SB&SL net. Then N is conservative and consistent.*

Unfortunately, the converse of this result is not true. The net of Figure 2.1b is an example. This net is conservative and consistent, but structurally non live. The question is, which condition should be added to conservativeness and consistency in order to get a characterization of SL&SB? We do not know the answer in general, but the main result of the next section gives the answer for FC nets.

2.2 A linear algebraic characterization of structural liveness and consequences

The 'missing condition' for FC nets is, maybe surprisingly, that the rank of the incidence matrix (i.e. the maximal number of linearly independent rows and columns) has to be a very simple function of the number of places, denoted by n, the number of transitions, denoted by m, and the number of arcs leading from a place to a transition, denoted by $a = |F \cap (P \times T)|$.

Theorem 2.3 (Rank Theorem) *Let N be an FC net. N is SL&SB iff it is conservative, consistent, and* $rank(C) = m - 1 - (a - n)$.

This result is illustrated in Fig. 2.1, 2.2 and 2.3. The net of Fig. 2.1.a satisfies the conditions, and is hence SL&SB (a live and bounded marking is shown). The other nets show that all conditions are necessary.

- Fig. 2.1.b: the net is FC, conservative and consistent but does not satisfy the rank equation. It is not SL.
- Fig. 2.2.a: the net is FC, conservative and satisfies the rank equation but it is not consistent. It is not SL.
- Fig. 2.2.b: the net is FC, consistent and satisfies the rank equation, but it is not conservative. It is not SB.

- Fig. 2.3.a: the theorem is false for Extended FC nets. The net is SL&SB, but does not satisfy the rank equation. Let us point only that the theorem could be reformulated in a less elegant way to make it hold for this subclass [CCS 90b].

- Fig 2.3.b: the theorem is false for non-FC nets. The net is SL&SB, but does not satisfy the rank equation.

This characterization was conjectured by the second author, together with J. Campos and G. Chiola, while studying performance bounds for LBFC stochastic systems. We will expose here part of the original argumentation [CCS 90a] that lead to one half of the property, since we think it can provide good insight on the result. Assume $\langle N, M_0 \rangle$ is an LBFC system, and a probabilistic prescription to solve the conflicts of N is given (e.g. if a place has three output transitions t_1, t_2, t_3, firing probabilities r_1, r_2, r_3 are associated to them with the constraint $r_1+r_2+r_3=1$). Let now G be a vector of dimension $|T|$ expressing the *relative frequency of firings* of the transitions in the steady-state of the net (the reader will have to believe that this steady state exists). After a little thought, it can be guessed that this vector is *unique*, once it has been properly normalized by setting the frequency of an arbitrary transition to 1. This means that G is completely characterized by

- The *structure* of the net N, that is represented by the incidence matrix C, provided there are no self-loops (i.e. $\forall t \in T \; Pre(t)^T \cdot Post(t) = 0$).

- The *probabilities* assigned to transitions in conflict. They can be represented by a matrix R, having a row for each pair of transitions in conflict and a column per transition.

For instance, consider the net of Fig. 2.1.a. It has one single pair of transitions in conflict, namely the pair formed by t_1 and t_2, the two output transitions of p_1. Thus, R has only one row. Let the conflict be solved with probability r for t_1, and $1-r$ for t_2. Then:

$$\frac{G(t_1)}{G(t_2)} = \frac{r}{1-r} \Leftrightarrow (1-r)G(t_1) - rG(t_2) = 0$$

Therefore $R = (1-r, -r, 0, 0, 0, 0, 0)$.
The fact that G is unique implies that, for any assignation of probabilities, the system

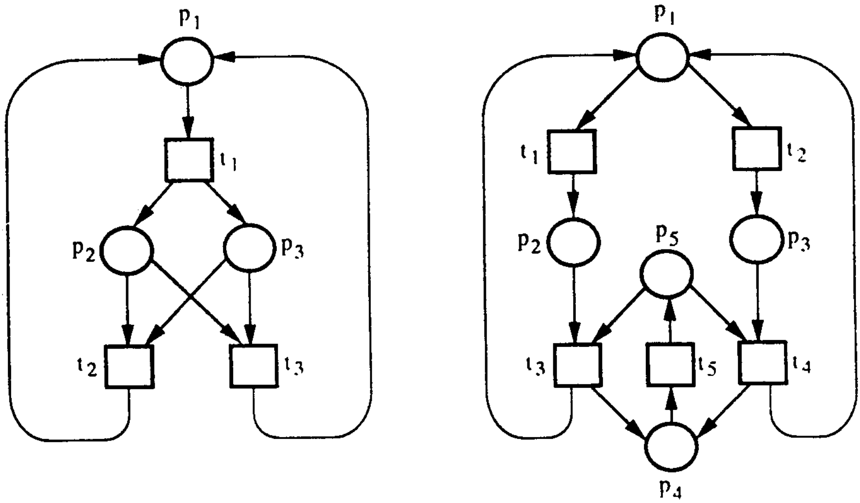
$$\begin{pmatrix} C \\ R \end{pmatrix} \cdot G_n = 0, G > 0$$

has a unique solution. This means that the space of right annullers of the matrix has dimension 1, and hence the dimension of the space generated by the rows of the matrix has dimension $m-1$. Since this happens for all the possible assignations of probabilities, the spaces generated by the rows of C and R must be *disjoints*, and hence

$$rank(C) + rank(R) = m - 1$$

Now, we can observe that the number of linearly independent rows of R is $a-n$: each place p contributes with $|p^\bullet| - 1$ independent constraints to R (e.g. if p has three output transitions t_1, t_2, t_3, only two probabilities r_1, r_2 can be set up independently, the third one being given by the constraint $r_1+r_2+r_3=1$). Adding up the constraints corresponding to all the places, this number is obtained. Hence, we have

$$rank(C) = m - 1 - (a - n)$$



(a) Extended free choice net
$r = \text{rank}(C) = 1$
$\rho = m+n-a-1 = 3+3-5-1 = 0$

(b) Asymmetric choice net
$r = \text{rank}(C) = 3$
$\rho = m+n-a-1 = 5+5-7-1 = 2$

Figure 2.3. Two live and structurally bounded nets with $r \neq$

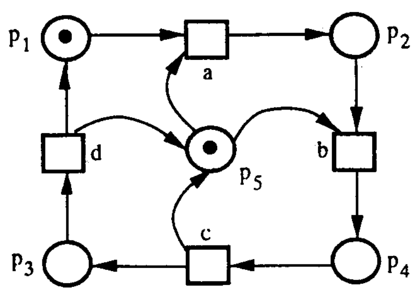


Figure 2.4. An SL&SB asymmetric choice net: It is non live even if all p-semiflows are marked.

In [Espa 90a] the algebraic characterization of structural liveness is proven for the particular case of *state machine decomposable free choice* (SMD-FC) nets. Independently of stochastic concepts, in [ES 90c] a formal proof of the theorem will be given. The necessary condition follows the above line of though, while the sufficiency is more complicated.

The first important fact about this theorem is that many of Hack's classical results can be derived from it or the proof process. In particular, the one which has an immediate proof is the so called "duality theorem". We need a previous definition: given a net $N = (P, T, F)$, its *reverse-dual* is the net $N_{rd} = (T, P, F^{-1})$ (i.e we replace places by transitions, transitions by places, and reverse the direction of the arcs). It is easy to see from the definition that if N is FC, then N_{rd} is FC as well.

We can now state the result.

Corollary 2.4 (Duality Theorem) *Let N be an FC net and N_{rd} its reverse-dual net. N is SL&SB iff N_{rd} is SL&SB.*

Proof: It follows easily from the definition of the reverse-dual net that $C_{rd} = -C^T$, where C and C_{rd} are the incidence matrices of N and N_{rd} respectively. Some consequences of this fact are:

(a) $rank(C) = rank(C_{rd})$

(b) N_{rd} is conservative iff N is consistent

(c) N_{rd} is consistent iff N is conservative

Using theorem 2.2, we have that N_{rd} is FC, conservative and consistent. Moreover

$$rank(C_{rd}) = rank(C) = n - 1 - (a - m) = m_{rd} - 1 - (a_{rd} - n_{rd}) = n_{rd} - 1 - (a_{rd} - m_{rd})$$

Hence, by theorem 2.3, N_{rd} is SL&SB. ∎

A second immediate consequence of theorem 2.3 is that SL&SB of an FC net is decidable in polynomial time.

Corollary 2.5 *Let N be an FC net. Then it can be decided in polynomial time if N is SL&SB.*

Proof: Linear programming problems have polynomial complexity [Karm 84, GT 89]. Conservativeness of a net can be decided by means of the following *linear programming problem* (LLP) (which is a little bit tricky, because the optimization function is identically zero)

$$\begin{array}{ll} \max & Y^T \cdot 0 \\ \text{s.t.} & Y^T \cdot C = 0 \\ & Y(i) \geq 1 \quad \forall i \in (1, n) \end{array}$$

A similar LLP can be used to decide consistency. Finally, the rank of a matrix can be calculated using standard methods of linear algebra. All these problems have polynomial complexity. Apply then theorem 2.3. ∎

2.3 A linear algebraic characterization of liveness for structurally live and structurally bounded free choice nets

Theorem 2.3 characterizes the lively and boundedly markable FC nets. Once we have one of these nets, we would like to know which are exactly the markings that make it live and bounded. By proposition 2.1, we know that the net is SL&SB, and hence bounded for any marking. It remains to characterize which markings make it live. The answer can be given in terms of the p-semiflows of the net, again a linear algebraic concept. A p-semiflow is *marked* at a marking M iff at least one of the places of its support is marked at M.

Theorem 2.6 [Espa 90b] *Let N be an SL&SB FC net. $\langle N, M_0\rangle$ is live iff all p-semiflows of N are marked at M_0 (i.e $\forall Y \gneq 0, Y^T \cdot C = 0: Y^T \cdot M_0 > 0$).*

The "only if" part holds in general: if a p-semiflow is unmarked at the initial marking, it remains unmarked at any reachable marking. But then the output transitions of the places of its support never fire. The "if" part can be proved from basic results in [Hack 72].

Theorem 2.6 is not true for non-FC nets. The net in Fig. 2.4 is SL&SB, and all its p-semiflows are marked at the initial marking shown. Nevertheless, the corresponding system is non-live.

A first corollary of theorem 2.6 is the well known result that the addition of tokens preserves liveness and boundedness in LBFC systems (this can be also directly derived from Commoner's Theorem, [Hack 72]), something that is not true for asymmetric choice nets (add a token to p_3 in Fig. 2.5).

Corollary 2.7 (Liveness Monotonicity) *Let $\langle N, M_0\rangle$ be an LBFC system. Then, for every $M_0' \geq M_0$, $\langle N, M_0'\rangle$ is live and bounded as well.*

Proof: By proposition 2.1, N is SL&SB. By theorem 2.6, M_0 marks all p-semiflows of N. Then M_0' marks them as well. Applying theorem 2.6 again, $\langle N, M_0'\rangle$ is live and bounded. ∎

A second corollary states that liveness and boundedness, as a whole, are decidable in polynomial time.

Corollary 2.8 (Polynomial Complexity) *Let $\langle N, M_0\rangle$ be an FC system. It can be decided in polynomial time if $\langle N, M_0\rangle$ is live and bounded.*

Proof: By proposition 2.1 and theorem 2.6, $\langle N, M_0\rangle$ is live and bounded iff N is SL&SB and all p-semiflows of N are marked at M_0. The first condition can be checked in polynomial time (corollary 2.5). The second condition can be checked solving, also in polynomial time, the following LPP:

$$\begin{array}{ll} \max & Y^T \cdot 0 \\ \text{s.t.} & Y^T \cdot C = 0 \\ & Y^T \cdot M_0 = 0 \\ & Y \gneq 0 \end{array}$$

It is obvious that all p-semiflows are marked iff this LPP has no solutions. ∎

This result can be compared with the one obtained by Jones, Landweber and Lien in [JLL 77]: deciding if an FC system is live is an coNP-complete problem.

To finish the section, let us remark that in [ES 89b] a quite different approach for deciding liveness and boundedness of FC nets, also polynomial, was presented. The method is based on an extension of Lautenbach's ideas relating deadlock, traps and p-semiflows [Laut 87b].

3 Analysis through reduction

The idea underlying reduction techniques is the following: given some properties to be analysed, transform the system into another one such that

- The properties hold in the transformed system if and only if they hold in the initial system. When this happens the transformation is said to *preserve* the properties.

- The properties are easier to analyse in the transformed system.

The transformations are performed by repeated application of a kit of *reduction rules* (elementary transformations) preserving the considered properties. A reduction rule consists on two parts:

- The *conditions* that have to be satisfied for the rule to be applicable

- The *changes* that specify the transformation

Both of them can be divided again into conditions and changes concerning the structure of the system and conditions and changes concerning its marking.

One of the problems of reduction analysis is that there can be systems to which the rules are not applicable. If these systems have a large size, their reachability analysis can be also computationally very complex. Here is where the notion of completeness of a kit of rules plays a rôle. A kit of rules is *complete* w.r.t. a class of systems if *all* the systems of the class satisfying the properties are transformed after the iterative application of the rules into one or more particularly simple systems (called *elementary systems*). In this case, we can decide if a system enjoys the properties just checking which is the final system after the transformation, and no further analysis is needed.

The goal of this section is to introduce two complete kits of reduction rules for the class of FC systems with respect to liveness and boundedness. In the first part of the section we introduce the two rules of the first kit. They are, essentially:

- Removal of so called marking structurally implicit places

- Substitution of certain P-graphs by a place

This kit can be hence considered as *place-oriented*. In the second part we introduce, making use of the duality theorem, a *transition-oriented* kit consisting of

- Removal of certain transitions, called structural bypasses

- Substitution of certain T-graphs by a transition

3.1 A place-oriented kit of reduction rules

Let us consider first the rule of removal of places. As usual, C denotes the incidence matrix of a net N. N^{-p} denotes the net obtained from N by removing the place p, together with its tokens and its input and output arcs. The corresponding incidence matrices are denoted by C and C^{-p} It is then clear that

$$\begin{bmatrix} C^{-p} \\ C(p) \end{bmatrix}$$

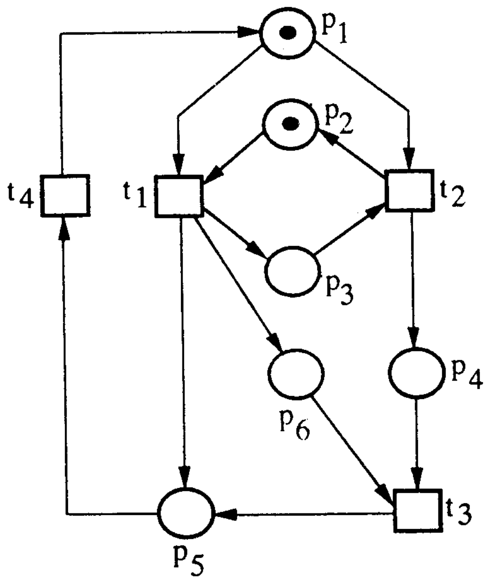


Figure 2.5. Marking liveness monotonicity does not hold for asymmetric choice nets.

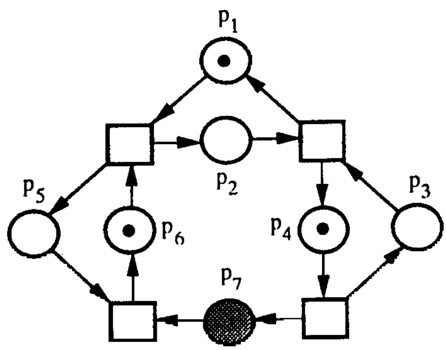


Figure 3.1. Place p_7 is 2-bounded, while the other places are safe.

where $C(p)$ is the row associated to p in C.

The places that can be removed are those whose rows in the incidence matrix are non-negative linear combinations of the rows of other places. We call them *marking structurally implicit places* (MSIPs for short) [CS 89b]. The reason of the name is given below.

Definition 3.1 *A place p is an MSIP of $N = (P, T, F)$ iff $C(p)$ is a positive linear combination of the rows of C^{-p}:*

$$\exists Y_p \gneq 0: \quad C(p) = Y_p^T \cdot C^{-p}$$

The following property can be easily derived from the definition.

Property 3.2 [Silv 85] *Let p be a MSIP. Then $\forall t \in {}^\bullet p$ $|t^\bullet| > 1$ and $\forall t \in p^\bullet$ $|{}^\bullet t| > 1$.*

This property provides an easy to check necessary condition for a place to be an MSIP. In particular, it is obvious that P-graphs are MSIP-free nets.

The reason of the name is that for every marking of the net N^{-p}, there is a marking of p that makes it *implicit*, meaning that the language of the net before and after adding this place does not change.

Property 3.3 [CS 89b] *Let p be an MSIP of N. Then, for every M_0^{-p}, there is M_0 (equal to M_0^{-p} in $P \setminus p$) such that the languages of $\langle N, M_0\rangle$ and $\langle N^{-p}, M_0^{-p}\rangle$ are equal.*

This property will be used later on in some proofs.

The following theorem shows that the removal of an MSIP from an FC net preserves SL&SB. The proof is given to show an application of the rank theorem.

Theorem 3.4 *Let $N = (P, T, F)$ be an FC net and $p \in P$ an MSIP of N. Then N^{-p} is SL&SB iff N is SL&SB.*

Proof: First, it is obvious that N^{-p} is FC with $n-1$ places, m transitions and $a-1$ input arcs to transitions. We show that, under the conditions above, N is conservative, consistent and $rank(C) = n - 1 - (a - m)$ iff N^{-p} is conservative, consistent, and $rank(C^{-p}) = (n-1) - 1 - ((a-1) - m) = rank(C)$. Applying then theorem 2.3, the result follows.

Let $Y_p \gneq 0$ be the vector such that $Y_p^T \cdot C^{-p} = C(p)$.

(i) N is consistent $\Leftrightarrow$ N^{-p} is consistent

($\Rightarrow$): Obvious, because no transition has been removed.

($\Leftarrow$): Since N^{-p} is consistent, there is $X > 0$, $C^{-p} \cdot X = 0$. We show that $C \cdot X = 0$, what implies that N is consistent as well.

$$C \cdot X = \begin{bmatrix} C^{-p} \cdot X \\ C(p) \cdot X \end{bmatrix} = \begin{bmatrix} C^{-p} \cdot X \\ Y_p^T \cdot C^{-p} \cdot X \end{bmatrix} = 0^T$$

(ii) N conservative $\Leftrightarrow$ N^{-p} is conservative

($\Rightarrow$): Since N is conservative, there is $W > 0$, $W^T \cdot C = 0$. This vector W can be written

$$W = \begin{bmatrix} Y \\ k \end{bmatrix}$$

with $k > 0$. Hence

$$0 = W^T \cdot C = Y^T \cdot C^{-p} + kC(p) = (Y + kY_p)^T \cdot C^{-p}$$

Since $Y + kY_p > 0$, N^{-p} is conservative.

($\Leftarrow$): Since N^{-p} is conservative, there is $W > 0$, $W^T \cdot C^{-p} = 0$. Take $\lambda > 0$ such that $Y' = \lambda W - Y_p \geq 0$. Then:

$$0 = \lambda W^T \cdot C^{-p} = (Y' + Y_p)^T \cdot C^{-p} = Y'^T \cdot C^{-p} + C(p) = [Y'^T \mid 1] \cdot C$$

Since $Y' > 0$, N is conservative

(iii) $rank(C) = n - 1 - (a - m) \Leftrightarrow rank(C^{-p}) = (n-1) - 1 - ((a - |p^\bullet|) - m)$

a) by property 3.2 and the FC definition, $|p^\bullet| = 1$. Hence

$$n - 1 - (a - m) = (n-1) - 1 - ((a-1) - m)$$

b) by the MSIP definition, $rank(C) = rank(C^{-p})$

and the result follows. ∎

It can be shown that the "if" part of this theorem holds in general. The "only if part" does not. The net of Fig. 2.5 is an example. The place p_3 is an MSIP of the net: $C(p_3) = C(p_6) + C(p_5) + C(p_1)$. Nevertheless, removing p_3 the net becomes structurally non live.

We already know that the removal of an MSIP preserves SL&SB for FC nets. We should see now which are the conditions for the removal to preserve liveness. It is not difficult to prove that, if every p-semiflow of N is marked at M_0, so is every p-semiflow of N^{-p} at the marking M_0^{-p}, consisting of the projection of M_0 on $P \setminus \{p\}$. Using then theorem 2.6, we obtain that if $\langle N, M_0 \rangle$ is live, so is $\langle N^{-p}, M_0^{-p} \rangle$. Unfortunately, the converse does not hold. It can be the case that, when removing p, we destroy unmarked p-semiflows of N, and pass from a non-live to a live system. That is why the rule requires that all p-semiflows of the source net have to be marked. The existence of an unmarked p-semiflow can be detected in polynomial time, by means of the LPP used in the proof of corollary 2.8. Fortunately, it is not necessary to check this condition every time we want to apply this rule, but *only the first one*. The reason is that, if the net contains an unmarked p-semiflow, it follows immediately that it is not live (the p-semiflow remains always unmarked, which implies that the output transitions of the places contained in its support can never fire). In this case the analysis is finished. On the other hand, if all the p-semiflows of the initial net are marked, this property is transmited to the reduced systems by the rule we introduce now, and the second one presented next.

REDUCTION RULE RIM

Structural conditions : N is an FC net containing an MSIP p

Marking conditions : Every p-semiflow of N is marked

Changes : Remove p with its tokens, input and output arcs.

Theorem 3.5 [Silv 85] **[ES 90a]** *RIM preserves liveness and boundedness, but not the bound of the net.*

The preservation of liveness and boundedness follows, essentially, from theorems 2.6 and 3.4. Figure 3.1 shows that the bound is not preserved.

Let us now introduce the second rule of this first kit, which consists of the substitution of a P-graph (N is a *P-graph* iff $\forall t \in T, |{}^\bullet t| = |t^\bullet| = 1$) by one single place. A previous definition is needed.

Let $N' = (P', T', F')$ be a subnet of $N = (P, T, F)$ [i.e. $F' = F \cap ((P' \times T') \cup (T' \times P'))$]. A place $p' \in P'$ is a *way-in* place of N' iff ${}^\bullet p \cap (T \setminus T') \neq \emptyset$, where the dot refers to N. Analogously, p' is a *way-out* place of N iff $p^\bullet \cap (T \setminus T') \neq \emptyset$. That is, the way-in places are those that can be used to "enter" into the subnet, and the way-out places the ones through which we can "get out" of it. *Way-in* and *way-out transitions* are defined analogously. They will be used later on.

Definition 3.6 *Let N' be a subnet of N. $N' = (P', T', F')$ is reducible to a place if:*

(a) N' is a P-graph containing at least one transition and $\forall t \in T' : |t^\bullet \cap P'| \leq 1$ and $|{}^\bullet t \cap P'| \leq 1$.

(b) For every $p' \in P'$, there exists at least an F'-path from a way-in place of N' to P'.

(c) For every $p' \in P'$ and every way-out place p'_o of N', there exists an F'-path from p' to p'_o.

The next definition, although somewhat complex, expresses no more than the standard notion of substitution of a net by a place.

Definition 3.7 *Let $\langle N = (P, T, F), M_0 \rangle$ be a system and $N' = (P', T'; F')$ a subnet of N reducible to a place. The net $N_r = (P_r, T_r; F_r)$, with*

- $P_r = (P \setminus P') \cup \{\pi\}$
- $T_r = (T \setminus T')$
- $F_r = (F \cap ((P_r \times T_r) \cup (T_r \times P_r))) \cup F_\pi$, *where*
  - $(t, \pi) \in F_\pi$ *iff there exists* $(t, p') \in F$ *with* $p' \in P'$
  - $(\pi, t) \in F_\pi$ *iff there exists* $(p', t) \in F$ *with* $p' \in P'$

is a macroplace reduction of N*, and* π *is the* macroplace *that replaces* N'*. The system* $\langle N_r, M_r \rangle$*, where* N_r *is the reduction of* N *and* M_r *is given by:*

- $M_r(p) = M_0(p)$ *if* $p \neq \pi$
- $M_r(\pi) = \sum_{p \in P'} M(p)$

is called a macroplace reduction of $\langle N, M_0 \rangle$.

REDUCTION RULE RMP

Structural condition : N contains a subnet N' reducible to a place.

Marking condition : none.

Changes : $\langle N, M_0 \rangle$ is reduced to $\langle N_r, M_r \rangle$, macroplace reduction in which N' is substituted by a macroplace

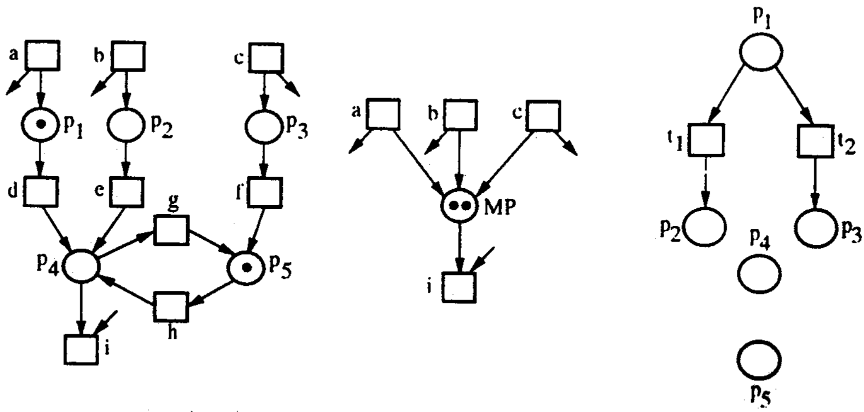
---

Figure 3.2 illustrates the macroplace reduction rule. In order to apply RDMP it is necessary to find the subnets of a net that can be reduced to a place. In [Silv 81] an efficient (polynomial) algorithm for this purpose is given. It consists of removing first all the transitions with more than one input or one output arc, what splits the net into one or more P-graphs (Fig. 3.2b is obtained from Fig. 2.1a removing t_3, t_4 and their incident arcs). Then simple recursive procedures are applied to each connected subnet to check conditions (b) and (c) of definition 3.6. The subset of places $\{p_1, p_2, p_3\}$ cannot be reduced to a single place because liveness would not be preserved (e.g. do this reduction in the context of Fig. 2.1b).

The utility of the macroplace concept lies in the following result, which holds in general.

Theorem 3.8 **[Silv 81]** *RMP preserves liveness and the bound of the system (thus boundedness).*

Figure 3.3 illustrates an application of the macroplace reduction rule. Let us see how we could go on reducing the net after that. The reader can easily check that $C(MP1) = C(MP2)$, and hence any of these two places is an MSIP. Removing any of them a strongly connected state machine is obtained. The application of the macroplace reduction rule leads to a net with one place and one transition, which is trivially live and bounded. A more complex reduction process is presented in Fig. 3.4. In the first step the macroplaces "B+D+F+G" and "M+J" are created. They can be removed one after the other, since both are MSIPs in the corresponding nets. After that, the macroplace rule can be used again, leading to the macroplaces "K+A+C" and "L+H+I". This last place is an MSIP. Removing it, and applying the macroplace rule again, a system with one place and one transition is obtained. Thus the original system was live and bounded. The procedure followed in these two examples is the following Reduction Algorithm.



(a) Reducible subnet and its reduction

(b) $\{p_1, p_2, p_3\}$ cannot be reduced to a single place

Figure 3.2. Macroplace reduction rule.

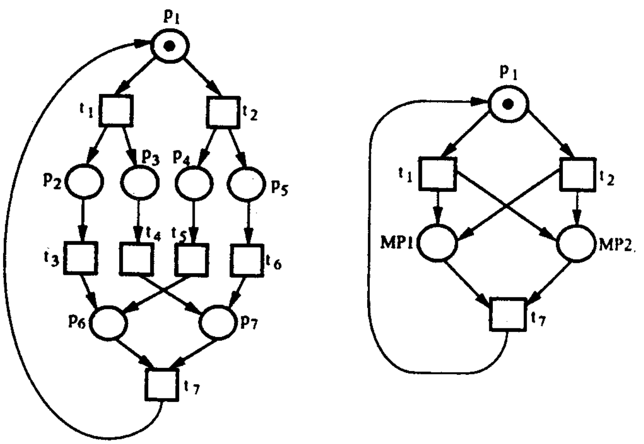


Figure 3.3. An application of macroplace reduction rule.

Reduction algorithm

```
begin
  input := < N, M_0 >, a FC system
  i = 0;
  < N_i, M_i > := < N, M_0 >;
  do while (< N_i, M_i > is reducible)
     do while (< N_i, M_i > is RMP-reducible)
        let < N_{i+1}, M_{i+1} > be the result of applying RMP to < N_i, M_i >;
        i := i + 1;
     od
     do while (< N_i, M_i > is RIM-reducible )
        let < N_{i+1}, M_{i+1} > be the result of applying RIM to < N_i, M_i >;
        i := i + 1;
     od
  od
  output < N_i, M_i >
end
```

The importance of this algorithm is that it is *complete*. In order to state this result we need first the concept of elementary system. A system $< N = (P, T, F), M_0 >$ is *elementary* iff $P = \{p\}$, $T = \{t\}$, $F = \{(p,t), (t,p)\}$ for some elements p, t and $M_0 > 0$.

Theorem 3.9 [ES 90a] (Soundness and Completeness of the Reduction Algorithm) *Let $\langle N, M_0 \rangle$ be an FC system. The application of the reduction algorithm to $\langle N, M_0 \rangle$ yields as output an elementary system iff $\langle N, M_0 \rangle$ is live and bounded.*

If we apply the above algorithm to any system constructed by marking the net in Fig. 2.1b, we see that the system cannot be reduced. Being not elementary, it follows that it is non-live or non-bounded.

We would like to expose briefly an outline of the completeness proof, because it provides good insight about how FC systems work. The proof relies heavily on Hack's decomposition result, whose statement requires some previous notions. A *P-component* of a net $N = (P, T, F)$ is a subnet $N' = (P', T', F')$ of N with the two following properties:

- N' is a strongly connected P-graph
- $^\bullet P' = P'^\bullet = T'$, where the dot refers to N

Notice that a P-component N' is characterised by the set of its places, P'. Moreover if $Y \in (0,1)^n$ with $Y(p) = 1$ iff $p \in P'$, Y is a p-semiflow of N (i.e. $Y^T \cdot C = 0$).

N is said to be *covered* by a set of P-components if every place belongs to at least one of the P-components of the set. This set is called a *cover*. The net of Fig. 3.4 is covered by the P-components with sets of places $\{A, B, D, F, G\}$, $\{A, C, E, K\}$, $\{H, I, E, L\}$ and $\{H, I, J, M\}$. Hack's decomposition theorem states that this is always the case for LBFC systems.

Theorem 3.10 [Hack 72] (Decomposition Theorem) *Let $\langle N, M_0 \rangle$ be an LBFC system. Then N can be covered by P-components.*

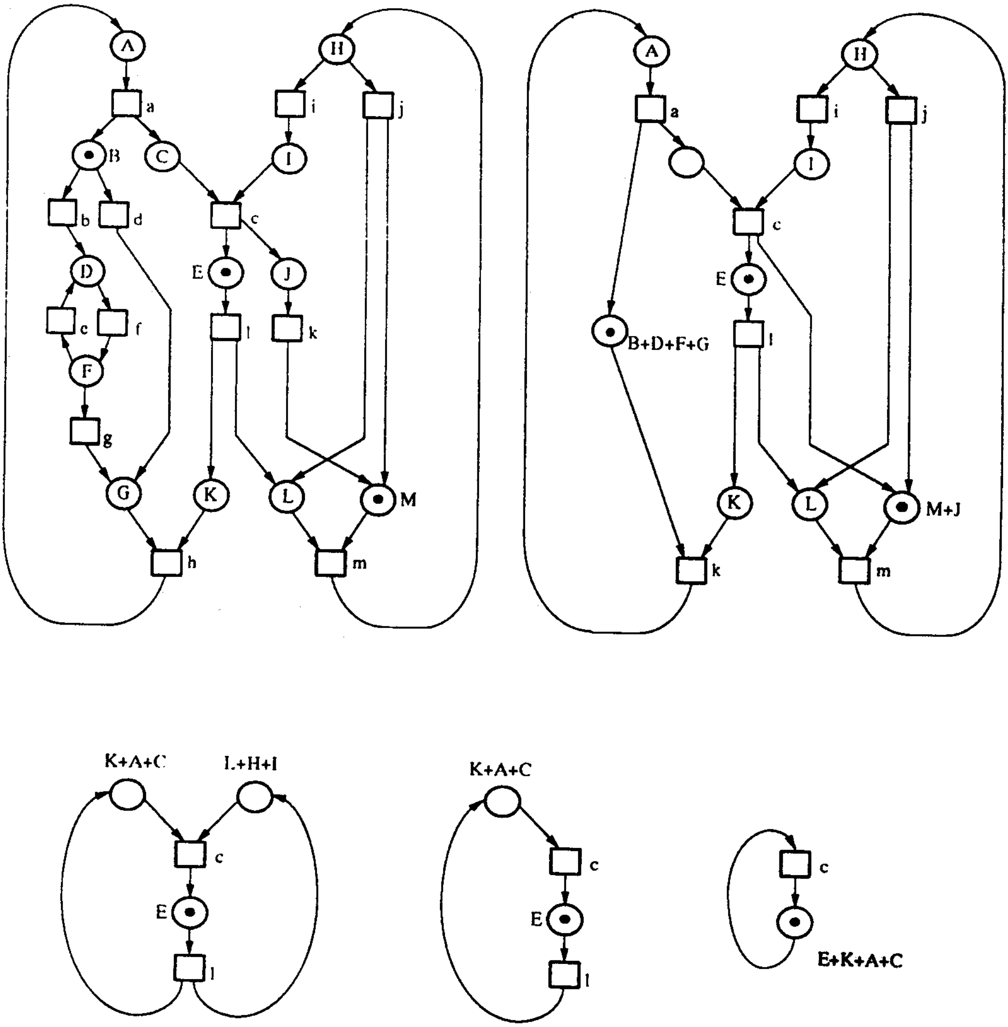


Figure 3.4. A reduction of a historical FC system (from [Hack 72]).

The completeness of the Reduction Algorithm is shown by proving the following result: it is always possible to apply the reduction rules to an LBFC system in such a way that the reduced system can be covered by a *smaller* number of P-components. Iterating this sequences of reductions, we get at the end a system covered by one single P-component, which implies that the system itself is a strongly connected P-graph. It is then possible to apply the macroplace rule to reach an elementary system.

We have to show hence how to reduce a system covered by r P-components to another one covered by $r-1$. We need a couple of previous notions.

Definition 3.11 *Let $C = \{N_1, \dots, N_r\}$ be a cover of P-components of N. The net $\hat{N}_1$ covered by $\{N_2, \dots N_r\}$ is called the* environment *of N_1.*

Let now N_1' be a subnet of N_1. N_1' is a private subnet *of N_1 iff the following three conditions hold:*

(a) N_1' is connected

(b) N_1' and $\hat{N}_1$ are disjoint

(c) N_1' is maximal, in the sense that no bigger subnet of N_1' satisfies both (a) and (b).

Private subnets are those parts of a P-component in which the environment does not interfere. In our example of Fig. 3.4, with the cover given above, the subnet with $\{B, D, F, G\}$ as places and $\{b, d, e, f, g\}$ as transitions is a private subnet of the P-component characterised by $\{A, B, D, F, G\}$.

Consider now a P-component such that when we remove all its private parts what remains (its environment) is strongly connected. It is not difficult to show that such a P-component always exists. In our example, the P-component generated by $\{A, B, D, F, G\}$ satisfies this condition.

We proceed in two steps. First, we show by means of the next theorem that the private subnets of such a P-component, let us call it N_1, can be reduced to a place. Then, we show that the macroplaces so obtained are MSIPs, and hence can be removed. After this, the reduced system is just the environment of N_1, which can be covered by $(r-1)$ P-components (the old cover without N_1).

Theorem 3.12 **[ES 90a] [Espa 90b]** *Let N_1 be a P-component of an LBFC system, such that its environment is strongly connected. Let N_1' be a private subnet of N_1. Then N_1' has exactly one way-out place.*

The only way-out place of the private subnet mentioned above is G. This theorem shows immediately that these private subnets fulfil the three conditions of definition 3.6. The third condition is immediately satisfied, because all places are connected to the only way-out place of the subnet, and hence to all of them. But the theorem has a clear interpretation as well. Private subnets of a P-component, which structurally can be considered as a sequential process, represent the part of the behaviour that the process can perform independently, without having to agree with other processes. Theorem 3.12 points out that this private behaviour is strongly constrained. Since private subnets have one single way out place, the environment, once a token has been put into the subnet, knows that eventually it will reach this place. The process can *delay* this final outcome, maybe for ever if the private subnets contains cycles and no fairness assumption is made, but cannot choose between different outcomes. Freedom of choice requires to pay a high price: *no process can take* *privately a decission which could have influence on the environment.* This result should not be surprising: if nobody can be forced to do anything that (s)he does not want to do, then nobody should be able to decide privately things that concern other people. From this point of view, theorem 3.12 just *formalises* this idea, giving a precise interpretation of the concepts *concern* and *privacy.*

The second part of our procedure requires to prove that the macroplaces we obtain after the reduction are MSIPs. This is done by means of the following result, which characterizes MSIPs in LBFC systems in terms of a surprisingly simple graph theoretical condition.

Theorem 3.13 [ES 90a] [Espa 90b] *Let $\langle N, M_0 \rangle$ be an SL&SB FC net and p a place of N. If N^{-p} is strongly connected, then p is an MSIP of N.*

As a last comment, the reader can check that both reduction rules can be applied by means of polynomial time algorithms. Because of the completeness of the reduction process, this provides an alternative polynomial algorithm to decide liveness and boundedness.

3.2 The reverse-dual kit

If we consider the structural parts of the place-oriented reduction rules (the structural conditions and the structural changes) we get *structural rules.* They transform nets, not systems, and *preserve the existence* of a live and bounded marking, instead of liveness and boundedness. In the case of FC nets, we know by proposition 2.1 that they preserve SL&SB as well. We can now profit from the duality theorem (corollary 2.4) to obtain what can be called structural *reverse-dual rules.* They are defined as follows:

- The strucural reverse-dual rule can be applied to N iff the structural rule can be applied to N_{rd}.
- If the structural rule transforms N into N', then the structural reverse dual rule transforms N_{rd} into N'_{rd}.

It is easy to see that if a structural rule preserves SL&SB, so does its structural reverse-dual rule (direct application of the duality theorem).

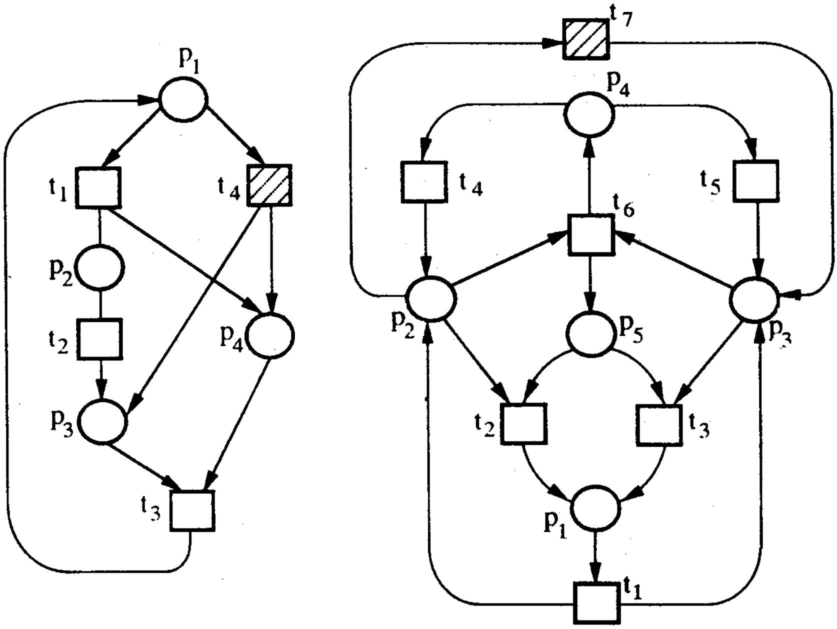
In order to get reverse-dual rules, acting on systems and preserving liveness and boundedness, we still have to care of the markings. We solve this problem using theorem 2.6.

Let us obtain the reverse-dual of the MSIP rule. If we removed places before, now we remove transitions, whose column in the incidence matrix is a positive linear combination of the columns corresponding to other transitions. We call these transitions *structural bypasses.* The reason is that their firing produces just the same effect than the firing of all the transitions present in the linear combination, each one as many times as the corresponding coefficient of the combination indicates. That is, firing this transition we *bypass* firing the other ones.

Let us now denote by N^{-t} the net obtained removing the transition t from N, together with its input and output arcs. The corresponding incidence matrices are denoted by C and C^{-t}. It is then clear that

$$C = [C^{-t}\ C(t)]$$

where $C(t)$ is the column associated to t in C.



(a) The original is a LBFC net: the addition of the structural bypass $t_4 = t_1 + t_2$ preserve structural liveness

(b) The original is a live and bounded P/T net. The addition of the structural bypass $t_7 = t_1 + t_2 + t_5 + t_6$ kill the net for any initial marking.

Figure 3.5. Structural bypass: positive linear combination of transitions.

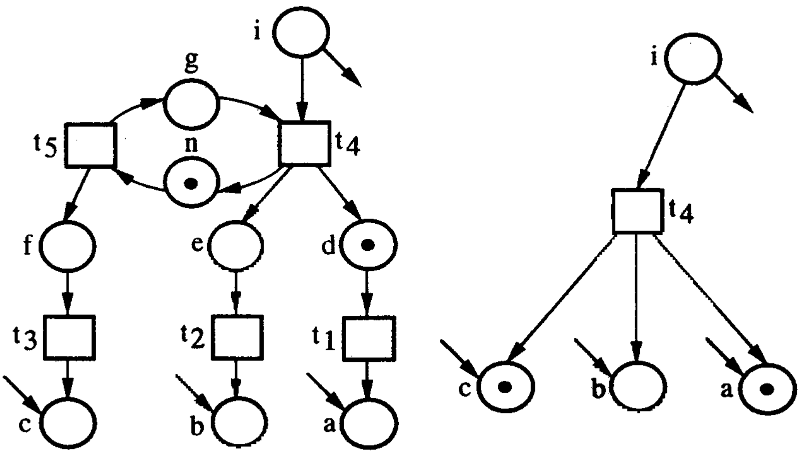


Figure 3.6. Macrotransition subnet reduction rule (ignoring the marking it is the reverse-dual schema of that in Fig. 3.2).

Definition 3.14 *A transition t is a structural bypass of the net $N = (P, T, F)$ iff $C(t)$ is a positive linear combination of the columns associated to other transitions:*

$$\exists X_t \gneq 0: \; C(t) = C^{-t} \cdot X_t$$

The two shaded transitions of Fig. 3.5 are structural bypasses.

Property 3.15 *Let t be a structural bypass. Then $\forall p \in {}^\bullet t: |p^\bullet| > 1$ and $\forall p \in t^\bullet: |{}^\bullet p| > 1$.*

The above property (dual of property 3.2) provides an easy to check necessary condition for a transition to be a structural bypass. In particular, it points out that T-graphs have no structural bypass.

The following result is easily obtained from the duality theorem and theorem 3.4.

Theorem 3.16 *Let t be a structural bypass of an FC net N. Then N is SL&SB iff N^{-t} is SL&SB.*

This theorem is not true in general. The structurally bounded net of Figure 3.5b is non-live for any marking, but removing the shaded structural bypass it becomes structurally live.

It can be proved that if t is a structural bypass then all the p-semiflows of N are marked iff all the p-semiflows of N^{-t} are marked. This shows that we do not need to care about the markings in order to preserve liveness and boundedness.

REDUCTION RULE RBY

Structural condition : N is an FC net containing a structural bypass t

Marking condition : none

Changes : Remove t with its input and output arcs.

Theorem 3.17 *RBY preserves liveness and boundedness, but not the actual bound of the system.*

Let us now introduce the reverse-dual of the macroplace rule. The next definition contains the reverse-dual concept of subnet reducible to a place. Way-in and way-out transitions are defined analogously to way-in and way-out places.

Definition 3.18 *Let N be an FC net and $N' = (P', T'; F')$ be a subnet of N (i.e. $F' = F \cap ((P' \times T') \cup (T' \times P'))$). N' is reducible to a transition if:*

(a) N' is a T-graph and $\forall p \in P': |p^\bullet \cap T'| \leq 1$ and $|{}^\bullet p \cap T'| \leq 1$

(b) For every $t' \in T'$, there exists at least an F'-path from t' to a way-out transition of N'.

(c) $\forall t' \in T'$, $\forall$ way-in transition t'_i of N': there exists an F'-path from t'_i to t'.

Figure 3.6 shows the macrotransition reduction rule. The intuition of the reduction process is straighforward and we skip to give a formal definition of the rule (in any case, is the reverse dual of that considered in definition 3.7). The reader can easily check that now T-graphs are reduced to a single macrotransition.

Figure 3.7 is self-explicative on an alternative reduction of the net in Fig.3.3a.

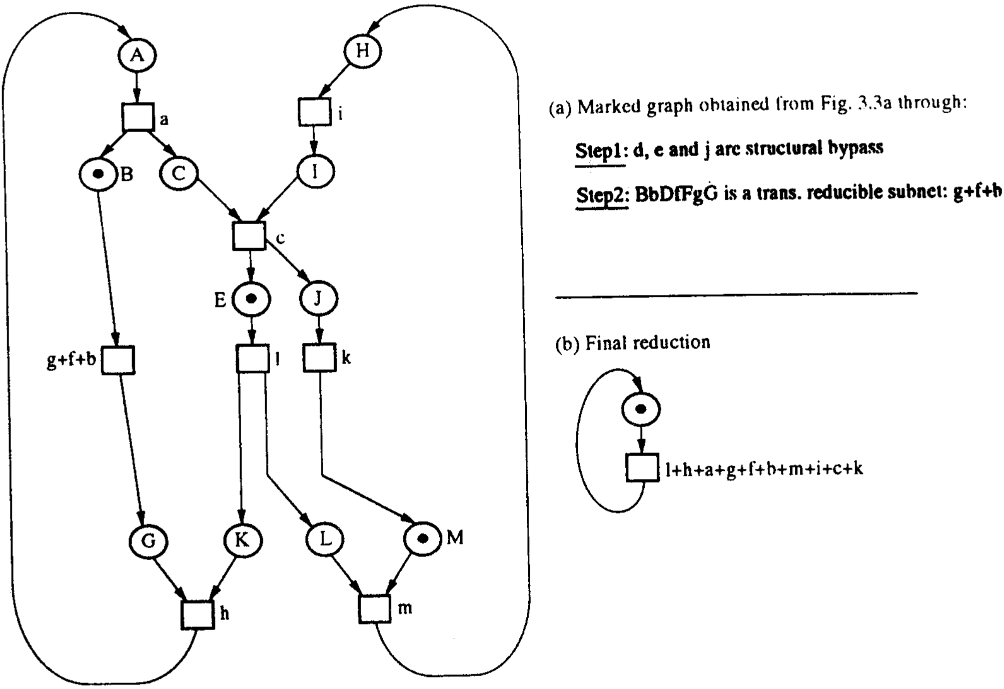


Figure 3.7. Structural bypass and macrotransition subnet reduction. of FC in Fig. 3.3a.

REDUCTION RULE RMT

Structure condition : N contains a subnet N' reducible to a macrotransition.

Marking condition : There is no unmarked p semiflow in N'.

Structural changes: • Substitute N' by the macrotransition.
Marking changes: • M_o^* is the restriction of M_0 to $P \setminus P'$.
• The final marking is obtained adding to each output place of a way-out transition the minimum number of tokens found in the different paths from the way-in transition.

The utility of the macrotransition concept lies in the following result.

Theorem 3.19 *RMT preserves liveness and boundedness, but not the actual bound of the system.*

Using the *duality theorem*, we can easily prove that this second kit of reduction rules is also *complete*. We have then provided two kits of reduction rules which characterize liveness and boundedness in FC systems. Since nothing prevents to interleave the applications of the four rules, faster (i.e. in less steps) reduction processes can be expected.

4 Top-down synthesis

Sections 2 and 3 have been devoted to analysis techniques. These techniques detect non-correct systems, but in general do not give any hint about how to proceed in order to improve the design.

This section and the next present an interesting alternative to this *trial and error* procedure based on *analysis and modification*: The use of *strict design methodologies*.

In these cases, the designer restricts him/herself to modifying and developing the model using only some very specific rules of *top-down transformation* and *composition* (modular approach), which can be safely applied because they are known to *preserve* the properties (here liveness and boundedness) desired for the system.

In the top-down design paradigm, to which this section is devoted, the synthesis procedure starts from an elementary one place-one transition system which is trivially live and bounded. This initial system is then enlarged in a stepwise way using the synthesis rules kit.

Synthesis (or design) rules are the reverse of reduction rules: instead of reducing the net system, a more detailed (enlarged) model is obtained. In section 3 a place-oriented and a transition-oriented complete reduction rules kits were presented. Their reverse will constitute *complete synthesis* rules kits: all LBFC systems can be generated stepwisely. Thus any of these synthesis kits provide an *alternative definition* of LBFC systems: instead of defining FC nets, and liveness and boundedness properties, LBFC systems are those that can be generated by means of rules of the identified synthesis kits. The important point with this idea in mind is that many net or system properties can be now proved in a relatively easy way by *inductive* reasoning: checking that the property is true for the elementary net/system and is preserved by any of the refinement rules in one of the kits.

4.1 The two synthesis kits

This section is devoted to the introduction of the two synthesis kits corresponding to the reduction kits introduced in the past section.

Place-oriented synthesis kit. This kit is composed by the reverse of the marking structurally implicit place (MSIP) and the macroplace reduction rules.

Let N_p be the net obtained adding a place to a net N. Given a marking M_{0p} of N_p, let M_0 be the marking obtained projecting M_{0p} on the places of N.

SYNTHESIS RULE SIM

Structural conditions : N is an FC net

Marking conditions : none

Structural changes: An MSIP p is added to N to yield an FC net N_p
(in particular, $|p^\bullet| = 1$)
Marking changes: • The marking of places of N remains unchanged.
• **if** $\exists Y \geq e_p$, $Y^T \cdot C_p = 0$ such that $Y^T \cdot M_{op} = 0$
then $M_{op}(p) > 0$
else $M_{op}(p) \geq 0$ **fi**

We can easily prove now the following result.

Property 4.1 *SIM preserves liveness and boundedness, but not the bound of the net.*

Proof: By theorem 3.4, N_p is SL&SB iff N is. Due to the nature of the marking changes, N_p contains no unmarked p-semiflow containing the new place p. This implies that N_p contains an unmarked p-semiflow iff N also contains one. Applying theorem 2.6, it follows that $\langle N_p, M_{0p}\rangle$ is live and bounded iff $\langle N, M_0\rangle$ is live and bounded. ∎

SIM, like RIM, does not preserve the bound of the system (Fig. 3.1) and does not preserve liveness for bounded asymmetric choice systems (Fig. 2.5).

Let us introduce now the macroplace refinement rule.

SYNTHESIS RULE SMP

Structural and marking conditions : none

Structural and marking changes : Transform $\langle N, M_0\rangle$ into $\langle \tilde{N}, \tilde{M}_0\rangle$, such that $\langle N, M_0\rangle$ is a macroplace reduction of $\langle \tilde{N}, \tilde{M}_0\rangle$

Property 4.2 *SMP preserves liveness and the bound of the system (thus boundedness).*

Proof: Follows easily from theorem 3.8. ∎

Transition-oriented synthesis kit. This kit is composed by the synthesis rules corresponding to the structural bypass and the macrotransition reduction rules.

Let N_t be the net obtained by adding transition t to N.

SYNTHESIS RULE SBY

Structural condition : N is FC

Marking condition : none

Structural change: N_t is an FC net obtained adding a structural bypass t to N (in particular, $|^{\bullet}t| = 1$)

Marking change: The old marking is preserved.

Property 4.3 *SBY preserves liveness and boundedness, but not the bound of the system. Nevertheless, if the system is live the bound is also preserved.*

Using the duality theorem and theorem 3.5, it follows that N_t is SL&SB iff N is SL&SB. The rest of the proof uses the following two facts: (1) the addition of a structural bypass preserves the p-semiflows, and (2) the behavioural bound of any place can be computed from the p-semiflows for LBFC systems [Espa 90b].

Let us now introduce the macrotransition refinement rule.

SYNTHESIS RULE SMT

Structural and marking conditions : none

Structural and marking changes : Transform $\langle N, M_0\rangle$ into $\langle \tilde{N}, \tilde{M}_0\rangle$, such that $\langle N, M_0\rangle$ is a macrotransition reduction of $\langle \tilde{N}, \tilde{M}_0\rangle$

Property 4.4 *SMT preserves liveness and boundedness, but not the actual bound of the system.*

The synthesis procedure. Any of the two refinement kits, place and transition-oriented, permit to construct all and only LBFC systems. This result, which follows easily from the completeness of their corresponding reduction kits, is formally stated next.

Theorem 4.5 *$\langle N, M_0\rangle$ is an LBFC system iff $\langle N, M_0\rangle$ is:*

a) an elementary system, or

b) the result of a finite sequence of transformations from a marked elementary system using the place-oriented or the transition oriented synthesis kits.

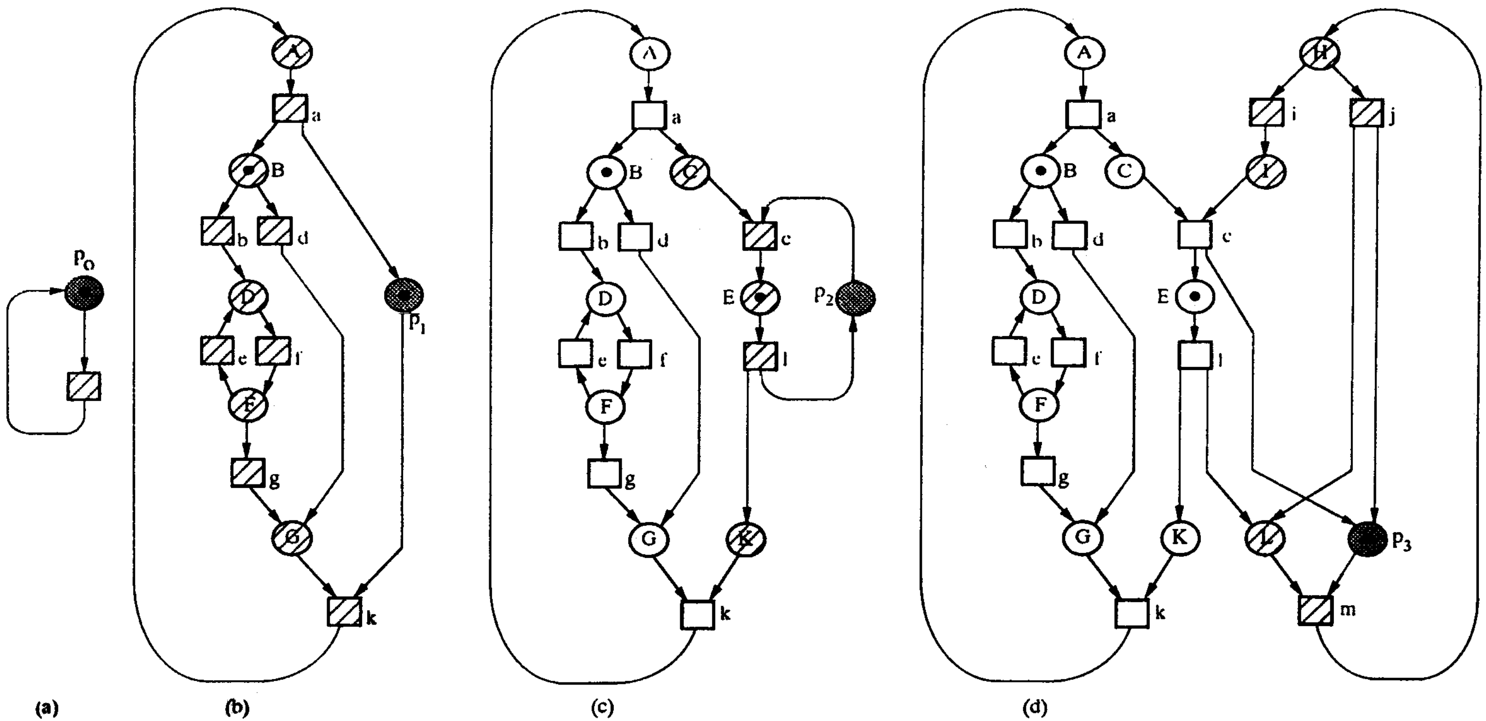


Figure 4.1. Top-down obtention of a system: The refinement of p_3 gives the Fig. 3.3a system.

Figure 4.1 shows a sequence of place-oriented synthesis. The initial place is refined into the shaded subnet. After that the MSIP p_1 is added. The refinement of p_1 leads to the shaded subnet in Fig. 4.1c. Later the MSIP p_2 is added. Its refinement leads to Fig. 4.1d, where the MSIP p_3 is added. Refinement of p_3 leads finaly to the net in Fig. 3.3a.

In the example of Fig. 4.1 places p_1, p_2 and p_3 when added are *implicit*: they do not change the behaviour of the model. In other words, these places being implicit do not constraint the firing language (i.e. the firing language is preserved) of the original net.

Let us suppose now that the elementary net (Fig. 4.1a) were marked with two tokens: $M_0(p_0) = 2$. If $M_0(p_1) = 1$ and $M_0(B) = 2$, the addition of p_1 does not kill the net (the new p-semiflow is obviously marked) but p_1 is no more an implicit place: p_1 constraints the behaviour. For example, transition k cannot be fired twice from M_0 without firing transition a.

4.2 Consequences of the completeness of the synthesis kits for the analysis of properties

Theorem 4.5 shows that LBFC systems can be defined *recursively* using any of the two refinement kits. Hence, if a property π is true for the elementary system chosen as *seed* of the synthesis procedure, and this same property is preserved for the two rules of one of the refinement kits, π is also true for all LBFC systems. We develop this idea using it to prove a couple of interesting results, whose proofs are only sketched. The first one is already known (see [BD 90]). The second can be deduced from Best/Voss/Vogler result on the existence of home states for LBFC systems [BV 84, Vogl 89] and proposition 2.1.

Proposition 4.6 (Relationship between T-components and minimal T-semiflows) *Let (N, M_0) be an LBFC system where $N = (P, T; F)$ and $X \geq 0$. X is a minimal T-semiflow iff the two following conditions holds:*

(a) $\forall t \in T : X(t) \in \{0, 1\}$

(b) There exists a T-component $N_1 = (P_1, T_1; F_1)$ of N such that the support of X (i.e. $\|X\| = \{t \in T : X(t) > 0\}$) is T_1, $\|X\| = T_1$.

Proof idea:
($\Leftarrow$): This part holds in general.
($\Rightarrow$): We prove this part by induction.
Base: The statement is trivially true for elementary systems.
Step: It is easy to see that the property is preserved by the macroplace rule, because the macroplace is substituted by a P-graph . We show now that the MSIP refinement rule preserves the property as well. In fact, if a vector X is a T-semiflow of the net before adding an MSIP, then it is also a T-semiflow of the net after adding it, because the MSIP place is a (positive) linear combination of rows in C, and no change can be produced on its right annulers.

Assume now we have a minimal T-semiflow satisfying the conditions of the theorem. By the induction hypothesis, to this T-semiflow corresponds a T-component. Moreover we know that this vector is also a T-semiflow of the final net. In particular, for SL&SBFC nets this means that the new place has as many input transitions in the support of the T-semiflow as output transitions: the number of both is exactly 0 (when the new place has no interference with the T-semiflow) or 1 (otherwise). In the first case, the old T-component is also a T-component of the new net. In the second case, the old T-component plus the new place is a T-component of the new net. ■

The second result requires to introduce the notion of *reversibility*. A system $\langle N, M_0 \rangle$ is reversible iff from every reachable marking M there is a firing sequence leading to M_0 (i.e. M_0 is a home state). As we have done with the notions of liveness and boundedness, we can also define *structural reversibility*. A net N is structurally reversible iff there exists an initial marking, M_0, such that $\langle N, M_0 \rangle$ is reversible.

Theorem 4.7 *Let N be an SL&SB FC net. Then N is structurally reversible.*

Proof: We generate inductively a reversible system $\langle N, M_0 \rangle$ using the place oriented kit.
Base: We start from the elementary system with one token on the place. This system is reversible.
Step: We use the two following particularizations of the two refinement rules.

i) The marking of the new MSIP places is large enough to make them implicit.

With this restriction, we ensure that the language of the net does not change. Let then $\langle N, M_0 \rangle$ be the old system and $\langle N_p, M_{0p} \rangle$ the new one, where p is implicit. Consider then $M_{0p}|\sigma\rangle M_p$. Since the language has not changed, $M_0|\sigma\rangle M$. By the induction hypothesis, there exists σ' such that $M|\sigma'\rangle M_0$. Since the language is preserved, $M_p|\sigma'\rangle M'_p$. It remains to show that $M'_p = M_{op}$. This can be done taking into account that the Parikh vector of the sequence $\sigma\sigma'$ is a T-semiflow of N. It was proved in the past result that if a vector is a T-semiflow in a net, it is also a T-semiflow after the addition of an MSIP. Hence $(\vec{\sigma\sigma'})$ is a T-semiflow of N_p, and $M_{0p}|\sigma\sigma'\rangle M'_p = M_{0p}$.

ii) The macroplace rule allows us to distribute arbitrarily the tokens of the substituted place on the new P-graph. Now we restrict this freedom, imposing that all the tokens have to be placed on the only way-out place of the new P-graph (see theorem 3.12).

Take now a reachable marking M of the system after substituting the macroplace. We sketch the procedure to find a firing sequence leading from M to M_0. The idea is the following: take the markings M'_0 and M' of the system before the P-graph is substituted which correspond to M_0 and M (that is, they are like M_0 and M, with the exception that all the tokens of the P-graph are now in the macroplace). By the induction hypothesis, there is a sequence $M'|\sigma'\rangle M'_0$. Now, every appearance of a transition in σ' that puts a token on the macroplace is substituted by a sequence composed by this transition and a firing sequence of transitions of the P-graph that put this token on the way out place. This way we produde a firing sequence σ of the system after the substitution. It is not difficult to see that $M|\sigma\rangle M_0$.

Therefore giving some restrictions on the markings of the place oriented kit only reversible systems are produced. Since we have not constrained the structural parts of the rules, we can still generate all the SL&SB FC nets with this new kit (although not all the LBFC systems). We have proved then that every SL&SB FC net can be endowed with a marking that makes it reversible. ■

LBFC systems are not reversible in general. An example is the net of Fig. 2.1a with initial marking $M_0^T = (0\ 1\ 0\ 0\ 1\ 0\ 0)$.

5 Modular synthesis

The design of large systems requires the use of *teams of designers*, each one in charge of a particular *subsystem* or *module*. The final system is built *composing* the subsystems. In this section we define two ways of composing subsystems and show how to interconnect them to preserve the properties of good behaviour that both the subsystems and the global system should enjoy.

Within our context, the above problem can be formulated in a very simple way: given several LBFC systems, characterize the compositions that preserve liveness and boundedness. We should warn the reader that we present a compositional solution of only the *structural part* of the problem. That is, we give exact conditions for the preservation of SL&SB under the compositions of nets we consider. Once we have obtained an SL&SB net, the initial markings making it live can be obtained applying theorem 2.6.

Since compositions of k nets can be splitted into $k-1$ compositions of 2 nets, we consider only this latter particular case.

5.1 Synchronizations and fusions

A very general notion of composition of two nets can be given as follows: a net $N = (P, T, F)$ is the *composition* of $N_a = (P_a, T_a, F_a)$ and $N_b = (P_b, T_b, F_b)$ iff N_a, N_b are subnets of N and $N = N_a \cup N_b = (P_a \cup P_b, T_a \cup T_b, F_a \cup F_b)$.

As an example, the net of Fig. 5.1.b is a composition of the two nets of Fig. 5.1a. An important notion for us concerning compositions is that of *interface*. The interface I between N_a and N_b in N is a subset of nodes defined as follows. A node x of N is in I iff:

- x is in both N_a, N_b
- There is at least one node of ${}^\bullet x \cup x^\bullet$ that is not in both N_a and N_b.

That is, we define the interface as the nodes where the two components "meet". Nodes "between" interface nodes need not be interface nodes themselves (e.g. Fig. 5.1c.2). In the example of Fig. 5.1a, the interface between the two components is formed by a place and a transition. The transition can be interpreted as a communication by *rendez-vous* between the two components, while the place corresponds to a communication by *shared states (common variables)*. Compositions in which these two types of communication mechanisms are present in the interface are difficult to interpret and lead to difficult to handle constructions. That is why we would like to consider compositions in which the interface is composed by only one type of nodes.

Definition 5.1 *Let $\{N_a, N_b\}$ be two nets. N is a* synchronization *of N_a and N_b iff*

$$p \in P_a \cap P_b \Rightarrow {}^\bullet p \cup p^\bullet \in T_a \cap T_b$$

(i.e. no place belongs to the interface).

N is a fusion *of N_a and N_b iff*

$$t \in T_a \cap T_b \Rightarrow {}^\bullet t \cup t^\bullet \in P_a \cap P_b$$

(i.e. no transition belongs to the interface).

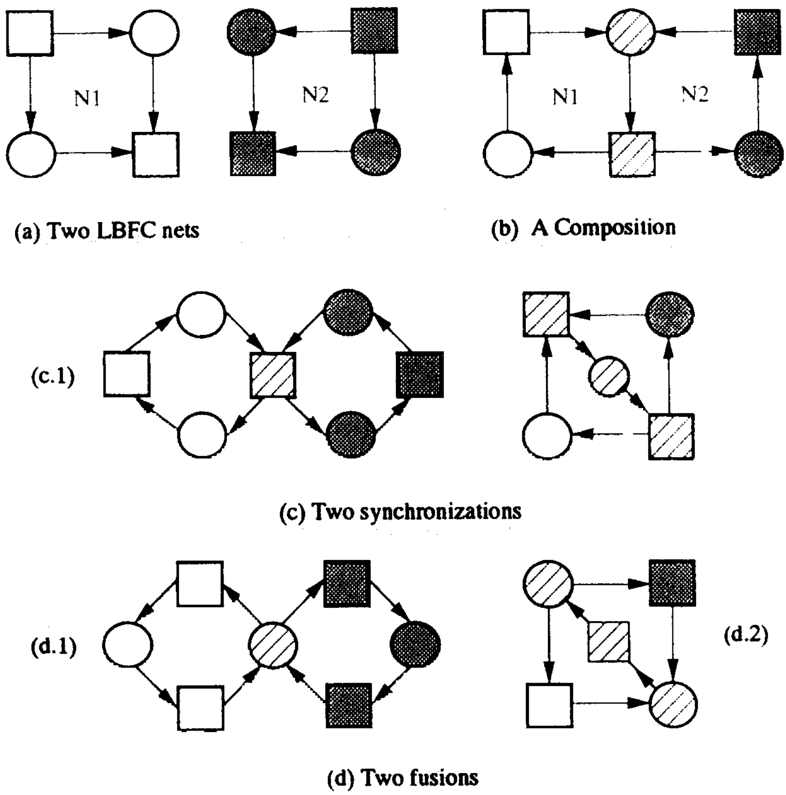


Figure 5.1. Composition, synchronization and fusion (the reverse-dual of synchronization) of free choice nets.

The nets of Fig. 5.1c are two different synchronisations of the nets of Fig. 5.1.a. The ones of Fig. 5.1d are two different fusions of the same nets.

We are interested on those synchronisations and fusions producing FC nets. We call them *FC-synchronisations (fusions)*. Obviously, the two components of an FC-synchronisation (fusion) must be FC. The net in Fig. 5.2 shows how the net in Fig. 3.3a can be obtained through FC-synchronizations.

Let us focus on the case of synchronisations. How to check that the synchronization of two SL&SB FC nets produces an SL&SB net? We could apply theorem 2.3 and check in polynomial time that the synchronisation is consistent and satisfies the rank equation (it can be shown that the synchronisation is conservative by construction). A possible question is: when the system is produced through several synchronisations, why not just check the final model? The answer is that, if we perform a check after each synchronisation, the possible design error is detected as soon as it is introduced. And this is particularly interesting in the context of FC nets, because the following *monotonicity* property can be proved.

Proposition 5.2 [ES 90b, Espa 90b] *Let N be an FC-synchronisation (FC-fusion) of $\{N_a, N_b\}$. N is SL&SB only if N_a and N_b are SL&SB.*

Hence, *if after a synchronisation of two conservative FC nets the composed net becomes non SL, it remains non SL.* Further synchronisations cannot *repair* design errors. Nevertheless, the reader can easily check (see Fig. 5.3) that this property does not holds when asymmetric choice nets are considered!

As a last remark, Fig. 5.4 shows that non-liveness of the FC-synchronized net can be originated on the initial marking obtained by the composition and not on the structure.

5.2 Interpreting FC-synchronization design errors

Theorem 2.3 can be used to detect when a bad composition was performed, but does not give information about the location and nature of the design error. We introduce in this section the results of [ES 90b] on this problem, which can be summarized as follows: the only possible design errors are two, called *synchronic mismatches* and *killing choices.*

The first structural design error: synchronic mismatches. Let us make first an informal introduction. Consider the two nets of the upper part of figure 5.5. They model the behaviour of John and Mary, two millionaires of Palm Beach. Every day John decides whether he well play tennis or not. If he does not play tennis, he goes dancing and then has a drink. If he does play tennis, then he is too tired to go dancing and just drinks. After the drink a new day comes and everything starts again.

Also Mary decides every day to play tennis or not. But, since she is in better shape than John, she always goes dancing after, and then has the drink. The question is: if John and Mary get married, and want to play tennis or not, go dancing and drink together, will the marriage eventually reach a deadlock? The marriage corresponds to the FC-synchronization at the bottom of the figure, and it is easy to see that the system will eventually deadlock. The reason is that John can execute the action "do play tennis" an arbitrarily large number of times without executing "go dancing", and Mary will be waiting to "go dancing".

To formalize the above problem, let us introduce a *synchronic relation*. Synchronic relations [Silv 87, SC 87] are tools of Synchronic Theory, which is a branch of net theory devoted to the study of dependences between the firings of transitions.

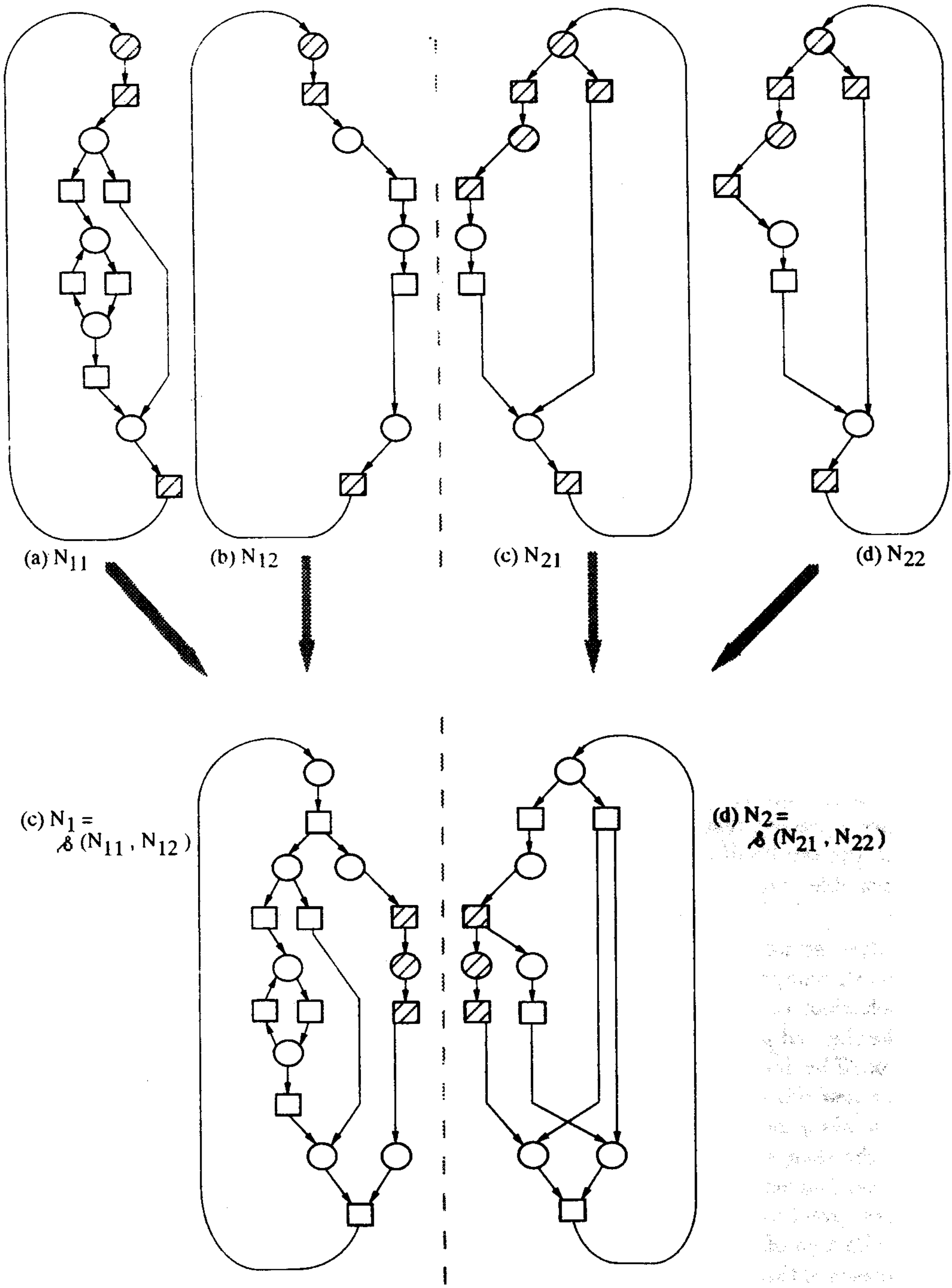


Figure 5.2. Modular composition (through synchronizations) of nets: The net in Fig. 3.3a is obtained synchronizing N1 and N2.

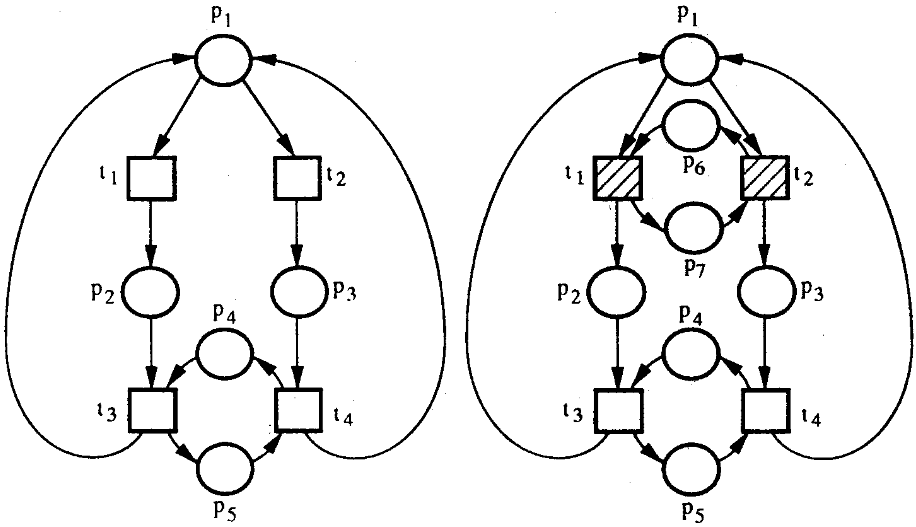


Figure 5.3. A structurally non-live FC net becomes a structurally live asymmetric choice net when the synchronization with a two place cycle is performed.

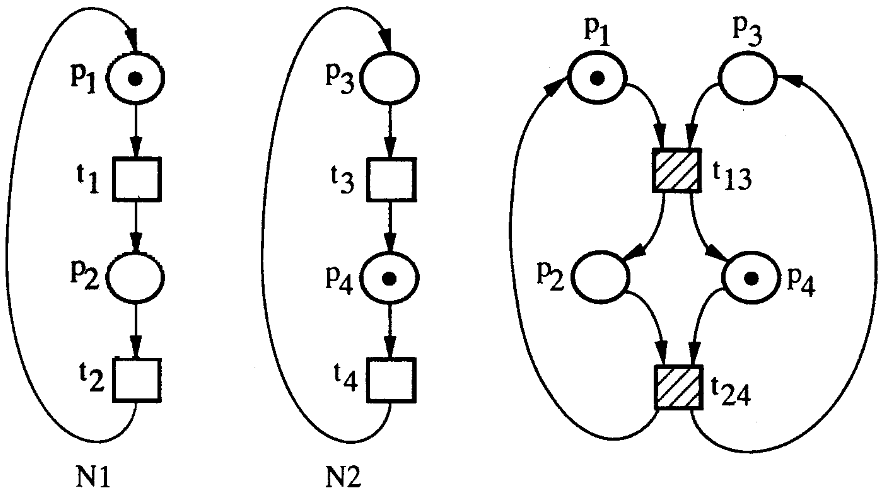


Figure 5.4. The synchronization of N_1 and N_2 leads to a structurally live net but non-live system (the cycle $p_3 - t_{13} - p_4 - t_{24}$ is unmarked!).

Definition 5.3 (Bounded Deviation, BD) *Let* $< N, M_0 >$ *be a system and* $R(N, M_0)$ *its marking reachability set.*

(1) $T_1, T_2 \subseteq T$ *are in* k-bounded relation *in the system iff* $\forall M \in R(N, M_0)$ *and* $\forall \sigma$ *applicable at* M *(i.e.* $M[\sigma >$*),* $\vec{\sigma}(T_2) = 0 \Rightarrow \vec{\sigma}(T_1) \leq k$.

(2) $T_1, T_2 \subseteq T$ *are in (behavioural)* bounded deviation relation *in* $< N, M_0 >$ *iff* $\exists k \in \mathbb{N}$ *such that* T_1, T_2 *are in k-bounded deviation relation.*

(3) $T_1, T_2 \subseteq T$ *are in* structural bounded deviation relation *in* N *iff* $\forall M_0, \exists k \in \mathbb{N}$ *such that* T_1, T_2 *are in k-bounded deviation relation.*

In the case of John and Mary, the two actions "do not play tennis" and "go dancing" are in structural BD-relation for John but not for Mary. That is, the synchronic relations of the two "partners" (subsystems) do not "match".

Definition 5.4 *Let* N *be a synchronization of* $\{N_a, N_b\}$. *The transitions* $t_i, t_j \in T_a \cap T_b$, *are a* synchronic mismatch *iff they are in structural BD-relation in one and only one of* N_a, N_b.

In our example, the two transitions corresponding to "do not play tennis" and "go dancing" constitute a synchronic mismatch.

Proposition 5.5 [ES 90b] *Let* N *be an FC-synchronization of* $\{N_a, N_b\}$, *where both* N_a, N_b *are SL&SB. If* N *contains a synchronic mismatch, then* N *is not structurally live.*

The second structural design error: killing choices. Let us go back to John and Mary. They have changed of hobbies, and like now to go to the cinema every day. There are two cinemas for millionaires in Palm Beach, the "Odeon" and the "Capitol". John decides each day which of the two cinemas he wants to go to, and so does Mary.

John and Mary want to get married and go to the cinema together, but both want to decide, without consulting the other, which of the two cinemas they will go to. The corresponding synchronisation is shown at the bottom of figure 5.6. Notice that the net contains no synchronic mismatches, but nevertheless leads to a deadlock for any marking. The deadlock is produced by the fact that the choices of John and Mary are *private*, but *concern the partner*. It is intuitively reasonable that these choices lead to non liveness for any marking. We call them *killing choices*.

Definition 5.6 *Let* N *be a FC-synchronisation of* $\{N_a = (P_a, T_a; F_a),\ N_b = (P_b, T_b; F_b)\}$. *A place* $p \in P_a$ *is a* killing choice *of* N_a *iff the following three conditions hold:*

(a) $p \notin P_b$

(b) There exists a T-component N_a^1 *of* N_a *containing* p *and a transition* $t_i \in T_a \cap T_b$.

(c) There exists an elementary path $B = (p, \ldots, t_j)$, $t_j \in T_a \cap T_b$, *such that* p *is the only node of* N_a^1 *in* B.

A killing choice of N_b *is defined analogously. It is said that* N *contains a* killing choice *iff it contains a killing choice of* N_a *or a killing choice of* N_b.

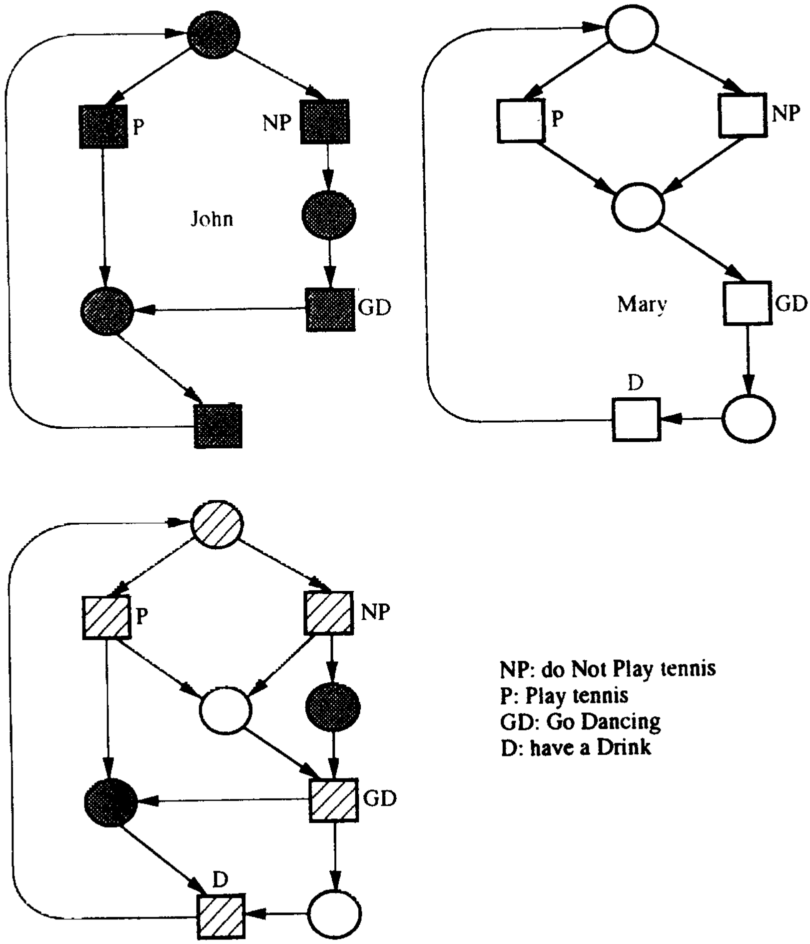
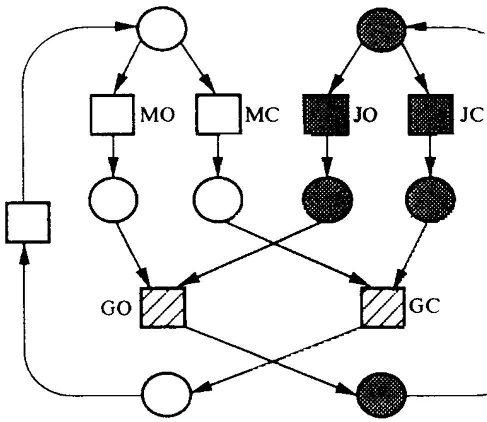
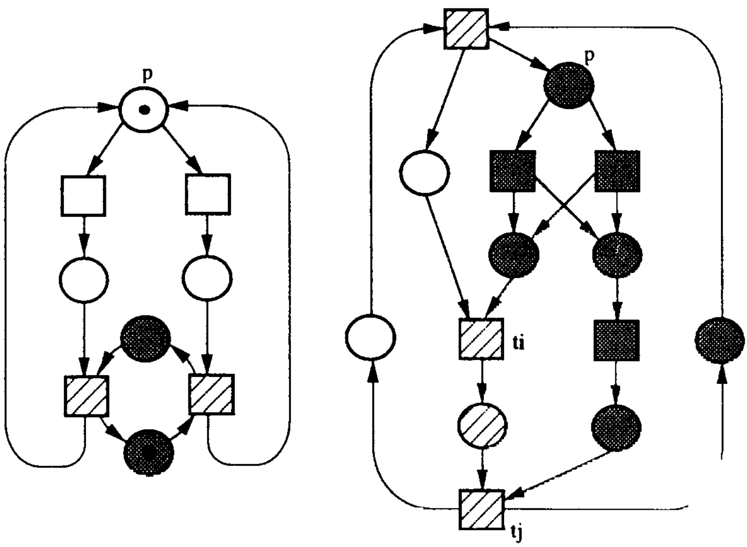


Figure 5.5. The transitions NP and GD are in synchronic mismatch.



MO / JO: Mary / John decides to go to the Odeon
MC / JC: Mary / John decides to go to the Capitol
GO: John and Mary go to the Odeon
GC: John and Mary go to the Capitol

Figure 5.6. Both components of the synchronization contain a killing choice.



(a) p is a killing choice (b) p is not a killing choice

Figure 5.7. Killing choices leads to deadlocks.

Notice that p is a place with more than one output transition, because it has at least one output transition in the T-component and another one out of it. In fact, N_a can decide freely at p whether the tokens are kept in the T-component or are taken out of it.

It could be thought at first sight that condition (b) of the definition is too complicated: apparently, in order to affect the behaviour of the other subnet, it would be sufficient the existence of two paths starting from the candidate to killing choice and ending at the two transitions t_i, t_j, paths with only the initial place in common. This is not enough, as the net in figure 5.7b shows. Place p seems to be a killing choice of N_b. Nevertheless, in spite of the existence of the two paths leading to t_i, t_j, the solution given by N_b to the conflict in p has no relevance for N_a. N_a only "sees" that N_b is always willing to fire both t_i and t_j, whichever was the branch selected by N_b at p. This is due to the fact that every T-component of N_b containing p and one of the transitions t_i , t_j contains also the other.

Proposition 5.7 **[ES 90b]** *Let N_a and N_b two SL&SB FC nets and N be an FC-synchronization of $\{N_a, N_b\}$. If N contains a killing choice, then N is structurally non live.*

This proposition is not true for asymmetric choice nets obtained by synchronization (Fig. 2.3b, with $t_i = t_3$ and $t_j = t_4$).

Completeness of the design errors. We hope that both killing choices and synchronic mismatches are intuitively seen as design errors, so it shouldn't be surprising that they lead to bad behaviours. What is not so intuitive is that every (structural) design error can be interpreted in terms of these two, or, in some sense, that these two are the only possible design errors.

Theorem 5.8 **[ES 90b]** *Let N_a and N_b two SL&SB FC nets and be N and FC-synchronization of $\{N_a, N_b\}$. N is structurally live iff it contains no synchronic mismatch and no killing choice.*

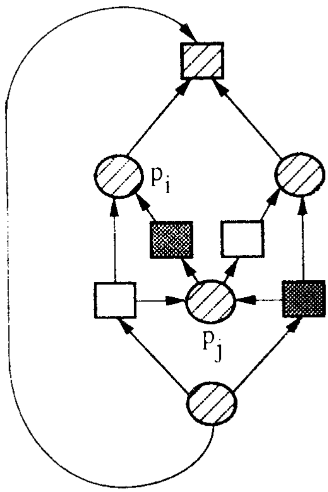
As a final remark, it can be pointed out that synchronic mismatches and killing choices can coexist in a bad design.

5.3 Fusions and design errors

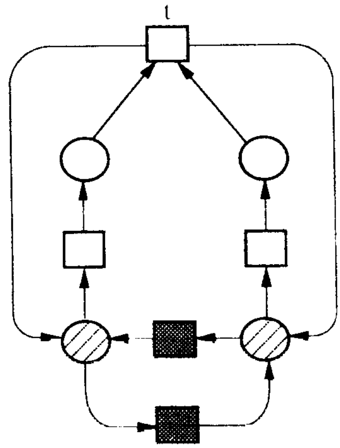
Applying the duality theorem we can obtain similar results to those of the past section about the reverse-dual concepts of synchronization, synchronic mismatch and killing choice. Due to lack of space we will not deal with them here.

Nevertheless, it is important to point out that *FC-fusions* (the reverse-dual of FC-synchronizations) are net compositions in which the interface is formed only by places. The reader is referred to Fig. 5.8 for illustrations of the *fusion* (or place) *mismatch* error, the reverse-dual of the synchronic mismatch, and the *killing joint* error, the reverse-dual of killing choice.

Once again, particular attention must be payed to the *completeness* of fusion mismatches and killing joints in order to explain all the possible errors. The reverse-dual of theorem 5.8 states that an FC-fusion of two SL&SB FC nets is SL&SB iff it contains no place mismatches and no killing joints.



(a) p_i and p_j represent a fusion (place) mismatch



(b) t is a killing joint

Figure 5.8. Fusion mismatch and killing joint design errors.

6 Conclusion

Arrived at this point. we should confess that our main goal has been to convince the reader of how nice, simple, powerful and computationally efficient the theory of LBFC systems can be. We have tried to state clearly concepts and results, illustrating them by means of examples and omitting the lenghty proofs. For more technical presentations the readers should consult the references. where the results are usually stated using quite different approaches. The cited works of Best. Commoner, Desel, Hack, Thiagarajan and Voss introduce many other beautiful results of the theory of Free Choice systems. In particular, it may be interesting to record two related recent results concerning home states and reachability in LBFC systems:

(1) In [BCDE 90], *home states* of LBFC systems are characterized as those for which all traps are marked.

(2) In [DE 90], the *reachability problem* is solved in polynomial time for reversible (i.e. M_0 is a home state) LBFC systems.

Free choice nets are rather limited for practical applications. For example, they cannot model systems with shared resources. On the other hand, as was pointed out in [Best 87], the research on FC systems has shown the existence of a big gap between them and asymmetric choice systems, which could appear to be the next natural class to consider. A possible solution to this conflict between the solutions offered by the theory and the requirements of practice could be the following: systems of practical interest are usually not FC, but we have observed that *they are often composed by subsystems which are FC*. These subsystems represent functional entities (i.e. the work we want to perform) which *compete* for resources or *cooperate* through message passing, modelled by means of *monitors* (a shared place implementing mutual exclusion mechanisms) and *buffers* (or mail boxes) respectively. We suggest *to extend the class of nets adding* some restricted communication mechanism like these mentioned here which, while representing a significant improvement in the expressive power, preserve some of the nice properties of FC systems. Some research is going on in this direction.

Acknowledgement

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References

[Bert 87] BERTHELOT, Transformations and Decompositions of Nets. In [BRR 87], *LNCS 254*, pp. 359-376.

[BRR 87] BRAUER, W.; REISIG, W.; ROZENBERG, G.; (EDS.): *Advanced Course on Petri Nets*, Lecture Notes on Computer Science 254 and 255, Springer-Verlag, Berlin.

[Best 87] BEST, E.: Structure Theory of Petri Nets: the Free Choice Hiatus. In [BRR 87], *LNCS 254*, pp. 168-205.

[BV 84] BEST, E.; VOSS, K.: Free Choice Systems Have Home States. *Acta Informatica 21*, pp. 89-100.

[BT 87] BEST, E.; THIAGARAJAN, P.S.: Some Classes of Live and Save Petri Nets. *Concurrency and Nets (Voss, K.; Genrich, H.J., Rozenberg, G., eds.)*, Springer-Verlag, Berlin, pp. 71-94.

[BD 90] BEST, E.; DESEL, J.: Partial Order Behaviour and Structure of Petri Nets. *Formal Aspects of Computing*, FACS-Vol 2, No. 2, pp. 123-138.

[BCDE 90] BEST, E.; CHERSAKOVA, L.; DESEL, J.; ESPARZA, J.: Characterization of Home States in Free Choice Systems. *Hildesheimer Informatik-Berichte Nr. 7/90* (July).

[CCS 90a] CAMPOS, J.; CHIOLA, G.; SILVA, M.: Properties and Performance bounds for closed free choice Synchronized Monoclass Queueing Networks. *Departamento de Ingeniería Eléctrica e Informática, Universidad de Zaragoza, Research Report 90.02*, January (27 pages).

[CCS 90b] COLOM, J.M.; CAMPOS, J.; SILVA, M.: On liveness analysis through linear algebraic techniques. *Departamento de Ingeniería Eléctrica e Informática, Universidad de Zaragoza, Research Report GISI 90.10*, June (17 pages)

[CHEP 71] COMMONER, F.; HOLT, A.W.; EVEN, S.; PNUELI, A.: Marked Directed Graphs. *Journal of Computer and System Sciences, Vol. 9, No. 2*, pp. 72-79.

[CS 89a] COLOM, J.M.; SILVA, M.: Convex geometry and semiflows in P/T nets. A comparative study of algorithms for computation of minimal p-semiflows. *Proceedings of the Xth International Conference on Application and Theory of Petri nets*, June, Bonn, pp. 74-95.

[CS 89b] COLOM, J.M.; SILVA, M.: Improving the linearly based characterization of P/T nets. *Proceedings of the Xth Int. Conf. on Application and Theory of Petri nets*, June, Bonn, pp. 52-73.

[Des 90] DESEL, J.: Reduction and Design of Well-behaved Concurrent Systems. *Proceedings of CONCUR'90 (Baeten, J.C.M.; Klop, J.W.; eds.)*. Amsterdam, August, LNCS 458, Springer-Verlag, Berlin, pp. 166-181.

[DE 90] DESEL, J.; ESPARZA, J.: Reachability in Reversible Free-choice Systems. *Technical University of Münich, SFB-Bericht Nr 342/11/90A*, June.

[Espa 90a] ESPARZA, J.: Synthesis rules for Petri Nets, and how they lead to new results. *Proceedings of CONCUR'90* (Baeten, J.C.M.; Klop J.W., eds.), Amsterdam, August. LNCS 458, Springer-Verlag, Berlin, pp. 182-198.

[Espa 90b] ESPARZA, J.: *Structure Theory of Free Choice nets*. Ph. D. thesis, Departamento de Ingeniería Eléctrica e Informática, Universidad de Zaragoza, June.

[EBS 89] ESPARZA, J.; BEST, E.; SILVA, M.: Minimal deadlocks in Free Choice Nets. *Departamento de Ingeniería Eléctrica e Informática, Research Report GISI 80.07*, March (16 pages). Also: *Hildesheimer Informatik Fachberichte, 89/1.*

[ES 89a] ESPARZA, J.; SILVA, M.: Circuits, Handles, Bridges and Nets.*Proceedings of the Xth Int. Conf. on Application and Theory of Petri nets*, June, Bonn, pp. 134-153.

[ES 89b] ESPARZA, J.; SILVA, M.: A polynomial time algorithm to decide liveness of bounded free choice nets. *Departamento de Ingeniería Eléctrica e Informática, Universidad de Zaragoza, Research Report GISI 89.04*, May (28 pages). To appear in *Theoretical Computer Science.*

[ES 90a] ESPARZA, J.; SILVA, M.: Top-down Synthesis of Live and Bounded Free Choice nets. *Proceedings of the XIth. International Conference on Application and Theory of Petri nets.* Paris, June, pp. - .

[ES 90b] ESPARZA, J.; SILVA, M.: Modular Synthesis of free-choice nets. *Departamento de Ingeniería Eléctrica e Informática, Universidad de Zaragoza, Research Report GISI 90.06*, March (29 pages).

[ES 90c] ESPARZA, J.; SILVA, M.: Free Choice nets, a Rank Theorem and its Consequences. To appear.

[Fink 90] FINKEL, A.: A minimal coverability graph for Petri Nets. *Proceedings of the XIth Int. Conf. on Applications and Theory of Petri Nets.* Paris, June.

[GL 73] GENRICH, H.J.; LAUTENBACH, K.: Synchronisationsgraphen. *Acta Informatica 2*, pp. 143-161.

[GT 89] GOLDFARB, D.; TODD, M.J.: Linear Programming. In *Optimization (G.L. Nemhauser et al eds.)*, North Holland, Amsterdam, pp. 73-170.

[Hack 72] HACK, M.H.T.: *Analysis of Production Schemata by Petri Nets.* Cambridge, Mass.: MIT, Dept. Electrical Engineering, MS Thesis (1972). Corrected June 1974

[Hill 85] HILLEN, D.: Relationship between Deadlock-freeness and Liveness in Free Choice Nets.*Newsletter of the GI Special Interest Group in Petri Nets and Related System Models, No. 19*, pp. 28-32.

[JLL 77] JONES, N.D.; LANDWEBER, L.H.; LIEN, Y.E.: Complexity of Some Problems in Petri Nets. *Theoretical Computer Science 4*, pp. 277-299.

[Karm 84] KARMARKAR, N.: A new polynomial-time algorithm for linear programming. *Combinatorica, Vol. 4*, pp. 373-395.

[Laut 87a] LAUTENBACH, K.: Linear Algebraic Techniques for Place/Transition Nets. In [BRR 87], *LNCS 254*, pp. 142-167.

[Laut 87b] LAUTENBACH, K.: Linear Algebraic Calculation of Deadlock and Traps. *Concurrency and Nets (Voss, K.; Genrich, H.J., Rozenberg, G.; eds.)*, Springer-Verlag, Berlin, pp. 315-336.

[MR 80] MEMMI, G.; ROUCAIROL, G.: Linear Algebra in Net Theory. *Net Theory and Applications. (Brauer, W.; ed.), LNCS 84*, Springer Verlag, Berlin, pp. 213-223.

[Mura 89] MURATA, T.: Petri Nets: Properties, Analysis and Applications. *Proceedings of the IEEE, Vol. 77, No. 4*, April, pp. 540-580.

[Sifa 78] SIFAKIS, J.: Structural Properties of Petri nets. *Mathematical Foundations of Computer Science 1978 (Winkowski, J.; ed.)*, Springer-Verlag, Berlin, pp. 474-483.

[Silv 81] SILVA, M.: Sur le Concept de Macroplace et son Utilisation pour l'Analyse des Reseaux de Petri. *RAIRO-Systems Analysis and Control, Vol. 15, No. 4*, pp. 335-345.

[Silv 85] SILVA, M.: *Las redes de Petri: en la Automática y la Informática.* Editorial AC, Madrid.

[Silv 87] SILVA, M.: Towards a Synchrony Theory for P/T Nets. *Concurrency and Nets (Voss, K.; Genrich, H.J., Rozenberg, G.; eds.)*, Springer-Verlag, Berlin, pp. 315-336.

[SC 87] SILVA, M.; COLOM, J.M.: On the computation of Structural Synchronic Invariants in P/T nets. *Advances in Petri Nets'87 (G. Rozenberg, ed.), LNCS 340*, Springer-Verlag, Berlin, pp. 306-417.

[TV 84] THIAGARAJAN, P.S.; VOSS, K.: A Fresh look at free Choice Nets. *Information and Control, Vol. 61, No. 2*, May, pp. 85-113.

[Vogl 89] VOGLER, W.: Live and Bounded Free Choice Nets have Home States. *Newsletter of the GI Special Interest Group in Petri Nets and Related System Models, No. 32*, pp. 18-21.