Bachelor Thesis - An Advanced Solver for Presburger Arithmetic

Description

Presburger arithmetic (also called the first-order theory of addition) is a language for expressing properties of (tuples of) natural numbers. Its elementary properties are linear equations and inequalities, which can be combined with the help of boolean operations and quantification.

Here is an example of a formula:

$$\exists y \exists z (x = 2y \wedge y = 3z) \wedge (x + 2y \leq 4) \wedge (y - 3x \leq -2)$$

Intuitively, the formula expresses that x and y are multiples of 2 and 3, respectively, and that they are solutions of the system of inequalities

$$\begin{array}{rcr} x + 2y & \leq & 4 \\ y - 3x & \leq & -2 \end{array}$$

It *solutions* are the pairs (x, y) satisfying the formula.

It is well known that the binary encodings of the set of solutions of a Presburger formula form a regular language. Using standard automata-theoretic constructions, one can construct a *solution automaton* recognizing this language.

The goal of the Bachelor Thesis is to develop and implement a solver for Presburger arithmetic, i.e., a program that accepts as input a formula of Presburger arithmetic, and outputs its solution automaton. On top of the basic functionality, the solver should incorporate a number of features:

- Use of advanced data structures.
  The size of the alphabet of the solution automaton grows exponentially in the number of variables. For this reason, automata labeled by binary decision diagrams should be used as data structure. This requires to extend the basic algorithms for manipulation of automata.

- User friendliness.
  Support for user-friendly description of formulas with macro options. Web interface.

- Basic visualization capabilities.
  Support for graphically displying fragments or projections of the solution space.

The implementation should allow future developers to easily extend or modify the functionality.

Further points that can be incorporated to thw work if time permits are:

- Support for computing the number of solutions, and optimal solutions.
- A dedicated solver for bounded Presburger arithmetic.

Prerequisites Basic knowledge of automata theory and logic is required. Having taken the course "Automata and Formal Languages" is an advantage.

Thesis The Thesis should clearly describe the problem and its solution, provide a clear pseudocode description of the algorithms used in the implementation, together with a brief justification of correctness, and describe the structure of the implementation in detail. It should also report on and discuss the performance of the implementation on an adequate set of examples.

After submitting the Thesis, the candidate must present his or her work in a talk open to all members of the Chair.

Supervision The work will be supervised by Javier Esparza and Jan Kretinsky.

Literature Wikipedia has a short article on Presburger arithmetic. The script of the course "Automata and Formal Languages" contains a Chapter on Presburger arithmetic, which describes a simple automata-based solver. The current version of the script can be obtained by sending an Email to Javier Esparza.