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Übungsblatt 4
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Übung zur Vorlesung Model Checking

1 CTL and LTL Specifications

(a) * Show that $\mathbf{A}f\mathbf{U}g$ is equivalent to

$$\neg[(\mathbf{E}(\neg g\mathbf{U}(\neg g \wedge \neg f)) \vee \mathbf{EG}\neg g].$$

(b) Given a formula $\mathbf{E}a\mathbf{U}b$, and a Kripke structure $K = (S, R, L)$, describe an algorithm which labels all states $s \in S$ where $K, s \models \mathbf{E}a\mathbf{U}b$, in **linear time**, i.e., in time $O(|S| + |R|)$.

Note: the algorithm has to label *all* states s where $K, s \models \mathbf{E}a\mathbf{U}b$, not just find some such states. For linear time, operations on lists, sets, etc. have to be counted.

(c) ** Same as above, for $\mathbf{EG}b$.

(d) * Let $K_1 = (S_1, R_1, L_1)$ and $K_2 = (S_2, R_2, L_2)$. We define $K_1 \leq K_2$ if $S_1 = S_2$, $R_1 \subseteq R_2$, and $L_1 = L_2$.

i) Show that $\leq$ is a partial order.

ii) Show the following lemma:
Let ϕ be an LTL specification, and $K_1 \leq K_2$ Kripke structures. If $K_2, s \models \phi$ then $K_1, s \models \phi$.

iii) Show that there exists a CTL specification which cannot be expressed in LTL.
Hint: Use the previous Lemma on the formula $\mathbf{EF}p$.

(e) ** Find a Kripke structure K, s such that $K, s \models \mathbf{AFG}p$ but $K, s \not\models \mathbf{AFAG}p$.

(f) *** Show that there exists an LTL specification which cannot be expressed in CTL.

2 Fixpoint Characterizations

Reconsider the Kripke structure in Figure 1. Using the fixpoint characterizations of CTL, show which states satisfy the following specifications:

(a) $\mathbf{EF}a$.

(b) $\mathbf{AG}a$.

(c) $\mathbf{E}a\mathbf{U}b$.

(d) $(a \vee q) \rightarrow \mathbf{EX}b$.

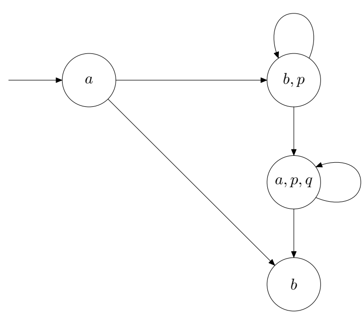

Figure 1: Kripke structure.