Technische Universität München
Institut für Informatik
Prof. Dr. Helmut Veith
Dipl.-Inf. Michael Tautschnig

SS 2007
Übungsblatt 2
Mittwoch, 16. Mai 2007

Übung zur Vorlesung Model Checking

1 Binary Decision Diagrams

(a) Describe BDDs for the Boolean functions which are always false and always true.

(b) Compute a BDD for the function $(a \vee b) \rightarrow (c \wedge b)$.

(c) Let $f(x_1, x_2, x_3, x_4)$ be a Boolean function expressing that the number $x_1x_2x_3x_4$ (in binary notation) is prime, and find a BDD for f.

(d) Consider the Boolean function $f(x_1, \ldots, x_4, y_1, \ldots, y_4)$ which expresses that the binary number $x_1x_2x_3x_4 + 1$ equals the binary number $y_1y_2y_3y_4$.

i) Describe f in propositional logic.

ii) Find a BDD for f with a good variable order.

iii) Generalize the BDD from 4 bits to arbitrary n.

(e) Same as above, with f describing $x_1x_2x_3x_4 < y_1y_2y_3y_4$.

(f) * Can problem 3 (primes) be generalized to arbitrary n? Why (not)?

2 CTL Specifications

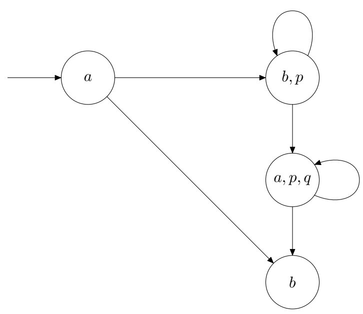

Figure 1: Kripke structure.

(a) On the Kripke structure in Figure 1, label the states according to the following specifications:

i) **EF**a.

ii) $\mathbf{AG}a$.

iii) $\mathbf{E}a\mathbf{U}b$.

iv) $\mathbf{AG}(p \rightarrow q)$.

v) $(a \vee q) \rightarrow \mathbf{EX}b$.

(b) Let p, q be atomic properties of systems. Express the following specifications in CTL as simply as possible: (There can sometimes be several possible solutions.)

i) p can never happen.

ii) p holds at least twice in the future (i.e., at two different points in time).

iii) p cannot hold for two time units.

iv) Whenever p holds, then q can not hold any more.

v) p holds until p becomes false.

vi) Either p holds in one step, or it will never hold.

vii) If it is possible to reach p at all, then p must be reachable infinitely often.

(c) Are the following formulas true, false, or neither ?

i) $(\mathbf{AG}p) \rightarrow (\mathbf{AG}\neg p)$.

ii) $(\mathbf{AG}p) \rightarrow (\mathbf{AG}p)$.

iii) $(\mathbf{AF}p) \rightarrow (\mathbf{EF}p)$.

iv) $(p \wedge \neg p) \leftarrow (q \wedge \neg p)$.

v) $(p \wedge \neg p) \rightarrow$ false.

vi) $(\mathbf{AX}p) \rightarrow (\mathbf{EF}p)$.

vii) $(\mathbf{AX}p) \rightarrow (\mathbf{EF}\neg p)$.

(d) Represent the following CTL formulas using only **EX**, **EU**, **EG**:

i) $\mathbf{EF}(s \wedge \neg r)$

ii) $\mathbf{AG}(r \rightarrow \mathbf{AF}ack)$

iii) $\mathbf{AGAF}e$

iv) $\mathbf{AGEF}r$

(e) For each of the formulas ϕ in the last two problems, describe two Kripke structures K_1, K_2 with initial states s_1, s_2, such that $K_1, s_1 \models \phi$ and $K_2, s_2 \not\models \phi$.