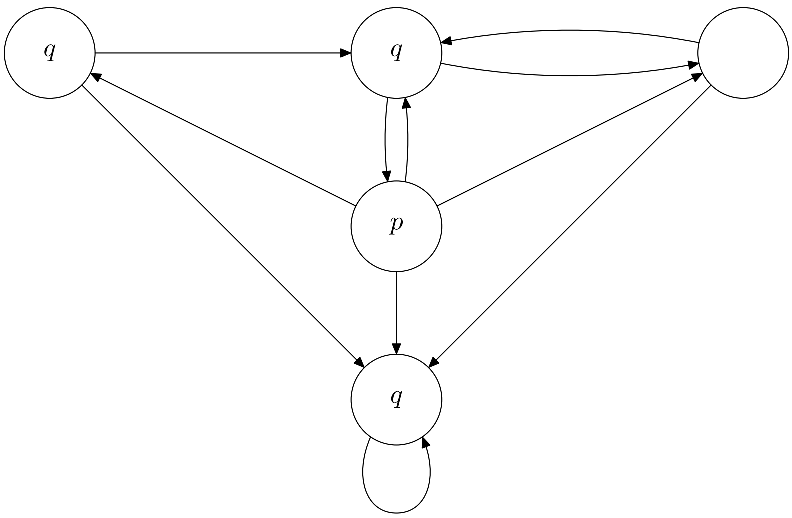
Problems and Exercises
"Model Checking", SS06
Part 2

Prof. Helmut Veith
Dipl.-Inf. Johannes Kinder
Dipl.-Ing. Robert Stepanek

Basic Fixpoint Theorems



1. By use of $\mathbf{EG}f_1 = \mathbf{gfp}Z(f_1 \wedge \mathbf{EX}Z)$ compute the set $\mathbf{EG}q$ for the Kripke structure shown above.

2. Consider the functions

$$H_1, H_2, H_3 : \mathcal{P}(\{1,2,3,4,5,6,7,8,9,10\}) \to \mathcal{P}(\{1,2,3,4,5,6,7,8,9,10\})$$

$$\begin{aligned} H_1(Y) &:= Y - \{1,4,7\} \\ H_2(Y) &:= \{2,5,9\} - Y \\ H_3(Y) &:= \{1,2,3,4,5\} \cap (\{2,4,8\} \cup Y) \end{aligned}$$

   (a) Show which of these functions are monotone.
   (b) Compute the least and greatest fixed points of H_3.
   (c) Does H_2 have any fixed points?

Binary Decision Diagrams

3. Consider the boolean functions $A := (a \wedge b) \rightarrow (c \vee b)$ and $B := (a \vee b) \rightarrow d$.

   (a) Compute an OBDD for A.

   (b) Compute the OBDD for B which has the same variable ordering as the OBDD for A.

   (c) Compute the OBDD for the boolean function defined by $A \wedge B$.

   (d) Same for $A \vee B$.

4. Let $f(x_1, x_2, x_3, x_4)$ be a Boolean function expressing that the number $x_1x_2x_3x_4$ (in binary notation) is *not prime*, and find a BDD for f. Can the problem be generalized to arbitrary n?

5. * Consider the Boolean function $f(x_1, \ldots, x_4, y_1, \ldots, y_4)$ which expresses that the binary number $x_1x_2x_3x_4 + 1$ equals the binary number $y_1y_2y_3y_4$.

   (a) Describe f in propositional logic.

   (b) Find a BDD for f with a good variable order.

   (c) Generalize the BDD from 4 bits to arbitrary n.

6. Let $f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4)$ be the Boolean function from above which expresses that the binary number $x_1x_2x_3x_4 + 1$ equals the binary number $y_1y_2y_3y_4$, and let A be the BDD for the function $g(x_1, x_2, x_3, x_4)$ where $g(0,0,0,0) = 1$, and $g(x_1, x_2, x_3, x_4) = 0$ otherwise. Compute the BDD for

   $$g' := \exists x_1, x_2, x_3, x_4 (f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4) \wedge g(x_1, x_2, x_3, x_4))$$

   and for

   $$g'' := \exists x_1, x_2, x_3, x_4 (f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4) \wedge g'(x_1, x_2, x_3, x_4)).$$

   [Hint: there is a *simple* way to do this!]