Problems and Exercises
"Nichtsequentielle Systeme und nebenläufige Prozesse", SS05
Part 2

Prof. Helmut Veith
Dipl.-Ing. Christian Schallhart
Dr. Stefan Katzenbeisser

Binary Decision Diagrams

1. Describe BDDs for the Boolean functions which are always false and always true.

2. Compute a OBDD A for the function $(a \wedge b) \rightarrow (c \vee b)$.

3. Compute a OBDD B for the function $(a \vee b) \rightarrow d$ with the same variable order as for A.

4. For the OBDDs A and B, compute the (a) conjunction and (b) disjunction.

5. For the OBDDs A and B, compute the OBDD for (a) A NAND B and (b) $\neg A$ AND B.

6. Let $f(x_1, x_2, x_3, x_4)$ be a Boolean function expressing that the number $x_1x_2x_3x_4$ (in binary notation) is *not prime*, and find a BDD for f. Can the problem be generalized to arbitrary n?

7. * Consider the Boolean function $f(x_1, \ldots, x_4, y_1, \ldots, y_4)$ which expresses that the binary number $x_1x_2x_3x_4+1$ equals the binary number $y_1y_2y_3y_4$.

   (a) Describe f in propositional logic.

   (b) Find a BDD for f with a good variable order.

(c) Generalize the BDD from 4 bits to arbitrary n.

8. * Same as above, with f describing $x_1x_2x_3x_4 < y_1y_2y_3y_4$.

9. ** Show that OBDDs are canonical: If OBDD A and OBDD B have the same variable order, and represent the same Boolean function, then they are isomorphic. Hint: Use induction.

10. Let $f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4)$ be the Boolean function from above which expresses that the binary number $x_1x_2x_3x_4+1$ equals the binary number $y_1y_2y_3y_4$, and let A be the BDD for the function $g(x_1, x_2, x_3, x_4)$ where $g(0, 0, 0, 0) = 1$, and $g(x_1, x_2, x_3, x_4) = 0$ otherwise. Compute the BDD for

$$g' := \exists x_1, x_2, x_3, x_4(f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4) \wedge g(x_1, x_2, x_3, x_4))$$

and for

$$g'' := \exists x_1, x_2, x_3, x_4(f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4) \wedge g'(x_1, x_2, x_3, x_4)).$$

[Hint: there is a *simple* way to do this!]

11. *^{5} **(optional)** Implement a simple OBDD library (in your favorite programming language) which provides
    - apply,
    - simplify,
    - equivalence checking and
    - pretty-printing (drawing with the help of some graph drawing software such as *dot*)