Problems and Exercises
“Nichtsequentielle Systeme und nebenläufige Prozesse”, SS05
Part 1

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1 CTL and LTL Specifications

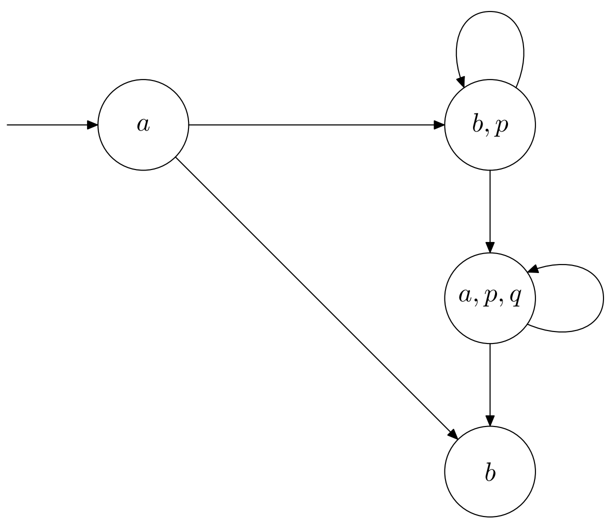

Figure 1: Kripke structure.

1. On the Kripke structure in Figure 1, label the states according to the following specifications:

(a) $\mathbf{EF}a$.

(b) $\mathbf{AG}a$.

(c) $\mathbf{E}a\mathbf{U}b$.

(d) $\mathbf{AG}(p \rightarrow q)$.

(e) $(a \vee q) \rightarrow \mathbf{EX}b$.

2. Let p, q be atomic properties of systems. Express the following specifications in CTL as simply as possible: (There can sometimes be several possible solutions.)

   (a) p can never happen.

   (b) p holds at least twice in the future (i.e., at two different time points).

   (c) p cannot hold for two time units.

   (d) Whenever p holds, then q can not hold any more.

   (e) p holds until p becomes false.

   (f) Either p holds in one step, or it will never hold.

   (g) If it is possible to reach p at all, then p must be reachable infinitely often.

3. Are the following formulas true, false, or neither ?

   (a) $(\mathbf{AG}p) \rightarrow (\mathbf{AG}\neg p)$.

   (b) $(\mathbf{AG}p) \rightarrow (\mathbf{AG}p)$.

   (c) $(\mathbf{AF}p) \rightarrow (\mathbf{EF}p)$.

   (d) $(p \wedge \neg p) \leftarrow (q \wedge \neg p)$.

   (e) $(p \wedge \neg p) \rightarrow \text{false}$.

   (f) $(\mathbf{AX}p) \rightarrow (\mathbf{EF}p)$.

   (g) $(\mathbf{AX}p) \rightarrow (\mathbf{EF}\neg p)$.

4. Represent the following CTL formulas using only **EX**, **EU**, **EG**:

   (a) $\mathbf{EF}(s \wedge \neg r)$

   (b) $\mathbf{AG}(r \rightarrow \mathbf{AF}ack)$

   (c) $\mathbf{AGAF}e$

(d) $\mathbf{AGEF}r$

5. For each of the formulas ϕ in the last two problems, describe two Kripke structures K_1, K_2 with initial states s_1, s_2, such that $K_1, s_1 \models \phi$ and $K_2, s_2 \not\models \phi$.

6. * Show that $\mathbf{A}f\mathbf{U}g$ is equivalent to

$$\neg[(\mathbf{E}(\neg g\mathbf{U}(\neg g \wedge \neg f)) \vee \mathbf{EG}\neg g].$$

7. Given a formula $\mathbf{E}a\mathbf{U}b$, and a Kripke structure $K = (S, R, L)$, describe an algorithm which labels all states $s \in S$ where $K, s \models \mathbf{E}a\mathbf{U}b$, in **linear time**, i.e., in time $O(|S| + |R|)$.

   Note: the algorithm has to label *all* states s where $K, s \models \mathbf{E}a\mathbf{U}b$, not just find some such states. For linear time, operations on lists, sets, etc. have to be counted.

8. ** Same as above, for $\mathbf{EG}b$.

9. * Let $K_1 = (S_1, R_1, L_1)$ and $K_2 = (S_2, R_2, L_2)$. We define $K_1 \leq K_2$ if $S_1 = S_2$, $R_1 \subseteq R_2$, and $L_1 = L_2$.

   (a) Show that $\leq$ is a partial order.

   (b) Show the following lemma:
   Let ϕ be an LTL specification, and $K_1 \leq K_2$ Kripke structures. If $K_2, s \models \phi$ then $K_1, s \models \phi$.

   (c) Show that there exists a CTL specification which cannot be expressed in LTL. Hint: Use the previous Lemma on the formula $\mathbf{EF}p$.

10. ** Find a Kripke structure K, s such that $K, s \models \mathbf{AFG}p$ but $K, s \not\models \mathbf{AFAG}p$.

11. *** Show that exists an LTL specification which cannot be expressed in CTL.